Supplementary Information: Range and strength of mechanical interactions of force dipoles in disordered elastic networks

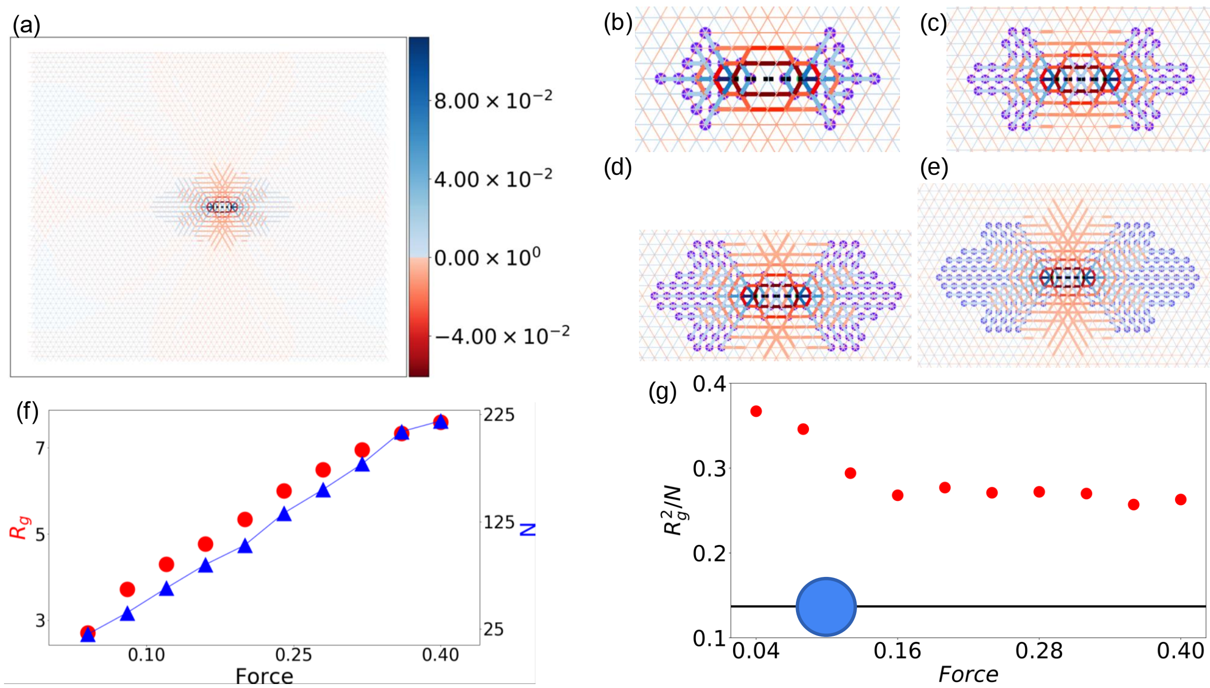

Fig. S1. **Strain clusters as measures of force transmission from a single dipole in a regular lattice.** (a) Representative network configuration and bond strains induced by a single dipole in an undiluted, regular triangular lattice ($p = 1$) for $f = 0.4$. The color bar shows the strain in the bonds, where blue (red) represents tension (compression). Thick bonds represent highly strained bonds that carry strain above the threshold value of $\epsilon_{th} = 0.003$. (b,c,d,e) Cluster of nodes (blue) connected to strongly tensile bonds (above strain threshold) for increasing values of dipole force: $f = 0.08, 0.16, 0.24$ and 0.4. (f) The size of each tensile cluster is characterized using two metrics: the number of nodes (N) in the cluster (blue data points), as well as its radius of gyration (red data points), R_g (eq. 2). Both measures show that the cluster size grows with increasing value of the dipole force. (g) Ratio of radius of gyration squared to number of nodes in tensile cluster, which is a measure of the deviation from circularity (or the anisotropy) of the cluster shape, as a function of the applied dipole force. For reference, the horizontal line is the expected value for a circular region in a hexagonal lattice.

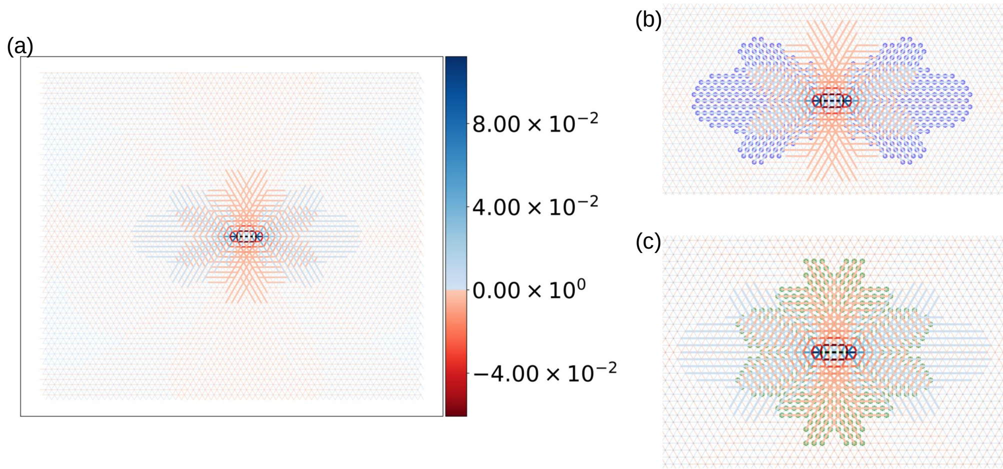

Fig. S2. **Strain plot and clusters with $\epsilon_0 = 0.0015$.** (a) The strain plot showing stretched and compressed regions. The tensile (shown in b) and compressive (shown in c) clusters, are both larger in size when we decrease the strain threshold, ϵ_0. However, there is no qualitative difference in using other values.

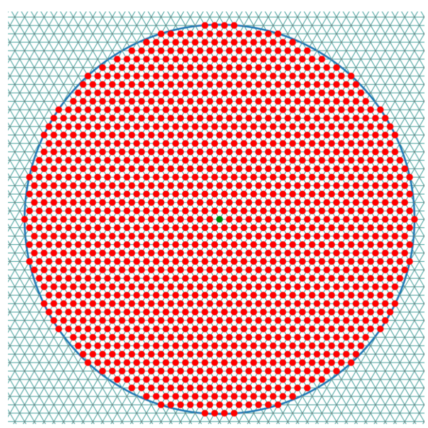

Fig. S3. **Circular region in triangular network to calculate the lower bound of shape parameter R_g^2/N.** Red nodes lie in a circular region of $r = 20$ around the central node shown in green. This region gives a value of $R_g^2/N = 0.1378$. Deviation from this value represents elongated cluster shapes.

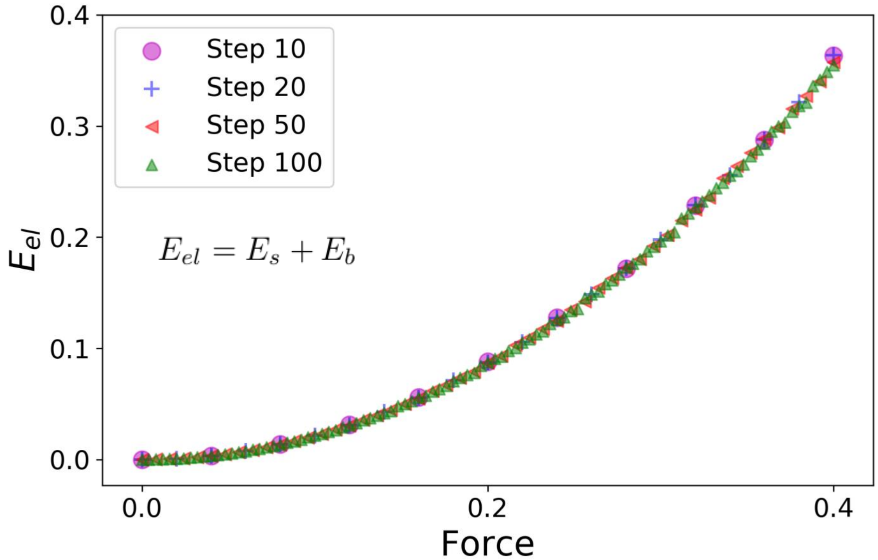

Fig. S4. **Elastic energy using different force step sizes.** Using different force steps, this plot shows the elastic energy of Network 1 with $p = 0.6$ and two dipoles separated along x axis with $d_x = 16$. Elastic energy is the sum of stretching and bending energies of the network. The agreement in energy values shows that the simulation is numerically accurate. For all the results in the paper, we took 10 incremental steps of equal $\Delta f = 0.04$ to reach the final force value of $f = 0.4$.

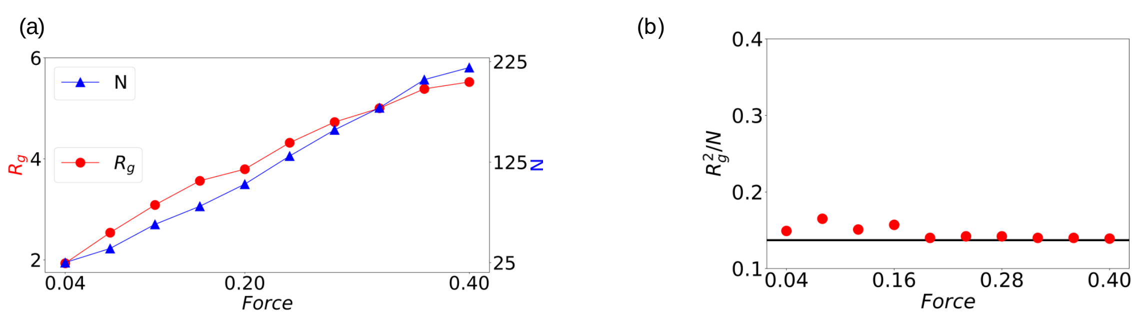

Fig. S5. **Compressive cluster size and shape versus force in a regular lattice.** (a) Increase in compressive cluster size with force ($p = 1$). Red circles are R_g values and blue triangles show N. (b) R_g^2/N at different fores show that the compressive clusters are very circular. The value for a circular cluster is shown by the thick black line.

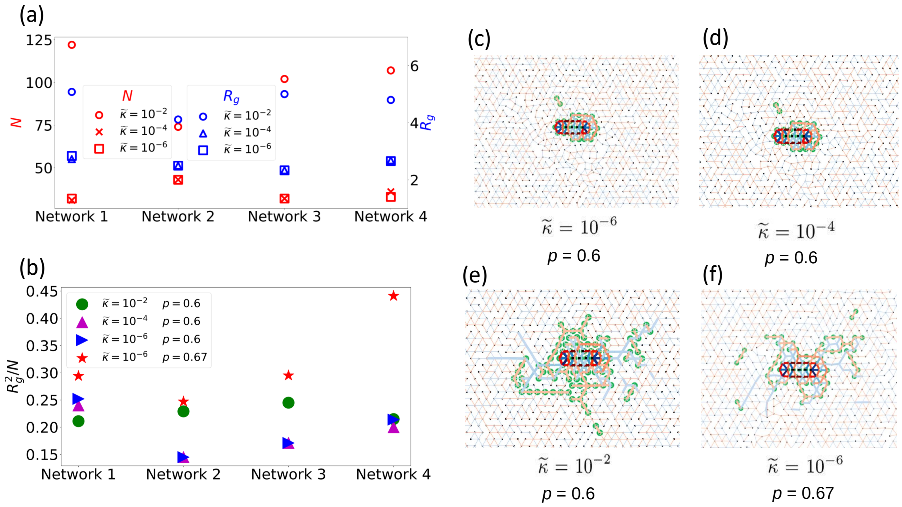

Fig. S6. **Compressive strain cluster around single dipole in disordered, bending-dominated networks.** All results (except inset of e) are for $p = 0.6$, which renders the network to be in the bending regime. (a) Similar to tensile clusters, number of nodes (and Radius of Gyration) in each network shows two distinct regimes: one for $\widetilde{\kappa} = 10^{-6}$ and 10^{-4}, and another for $\widetilde{\kappa} = 10^{-2}$. (b) For networks 2 and 3, a similar result as (a) is found. However, networks 1 and 4 show that clusters for $p = 0.6$ and $\widetilde{\kappa} = 10^{-2}$ have similar values as that for networks with $\widetilde{\kappa} = 10^{-6}$ and 10^{-4}. This can be attributed to the fact that when clusters are small (for example, c,d show Network 3), a small amount of anisotropicity significantly increases the value of R_g^2/N. Although it is visually evident that the cluster formed when $\widetilde{\kappa} = 10^{-2}$ (e) is much larger and different than those in (c,d), the values of the shape parameter are similar. (f) The compressive cluster with $p = 0.67$ and $\widetilde{\kappa} = 10^{-6}$ is similar to that of $p = 0.6$ and $\widetilde{\kappa} = 10^{-2}$ - suggesting that these networks lie in the bend-stretch coupled regime. Here the forces percolate farther than for networks in bending regime. This again confirms our previous assertion about accessing the special bend-stretched coupled regime in two ways.

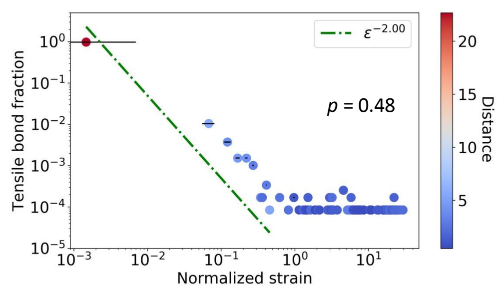

Fig. S7. **Tensile strain distribution for networks with $\mathbf{p = 0.48}$.** This plot combines all bonds of the four networks at $p = 0.48$. The strain histogram has a gap at intermediate strain values for networks that lie in bending regime. Almost all tensile bonds are not particularly strained. A very small number of tensile bonds have high strain values.

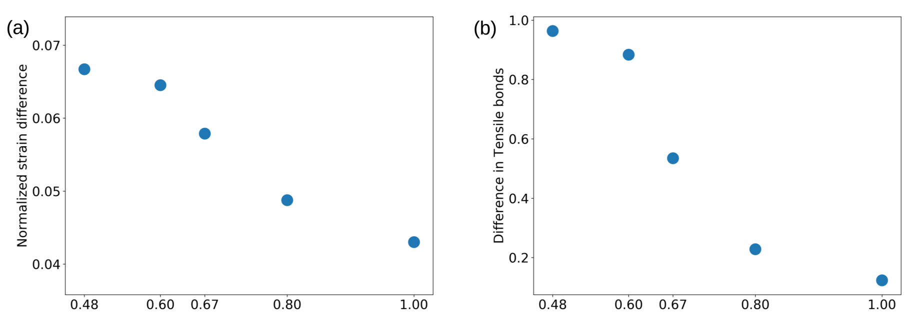

Fig. S8. **Difference in normalized strains and frequency for first two bins of strain histograms.** (a) Difference in normalized strain mean values between the first two bins of histograms in Fig. 6 and Fig. S7 . The difference increases with dilution. (b) Difference in normalized frequency of the first two bins of histograms in Fig. 6 and Fig. S7. Here again, the difference increases with dilution.

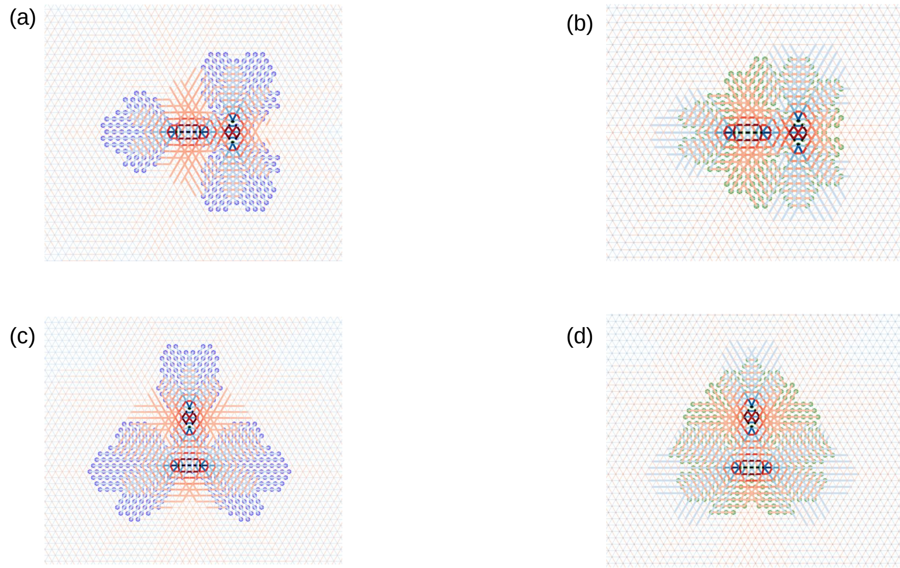

Fig. S9. **Strain clusters for dipoles that are perpendicular to each other.** (a,b) Tensile and compressive clusters for dipoles which are perpendicular to each other and at a distance of $d_x = 6$ from each other. (c,d) Tensile and compressive clusters for dipoles that are perpendicular to each at distance of $d_y = 8$ rows.

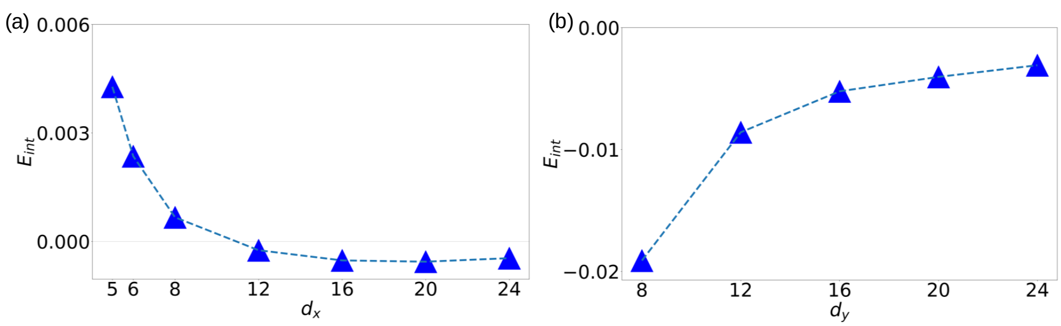

Fig. S10. **Interaction energy of two dipoles oriented along y axis.** (a) The interaction energy of two dipoles oriented along y axis and separated along x axis decreases in magnitude with increasing separation. (b) Two dipoles oriented along y-axis and separated along y axis have a negative interaction energy that decreases in magnitude with increasing separation.

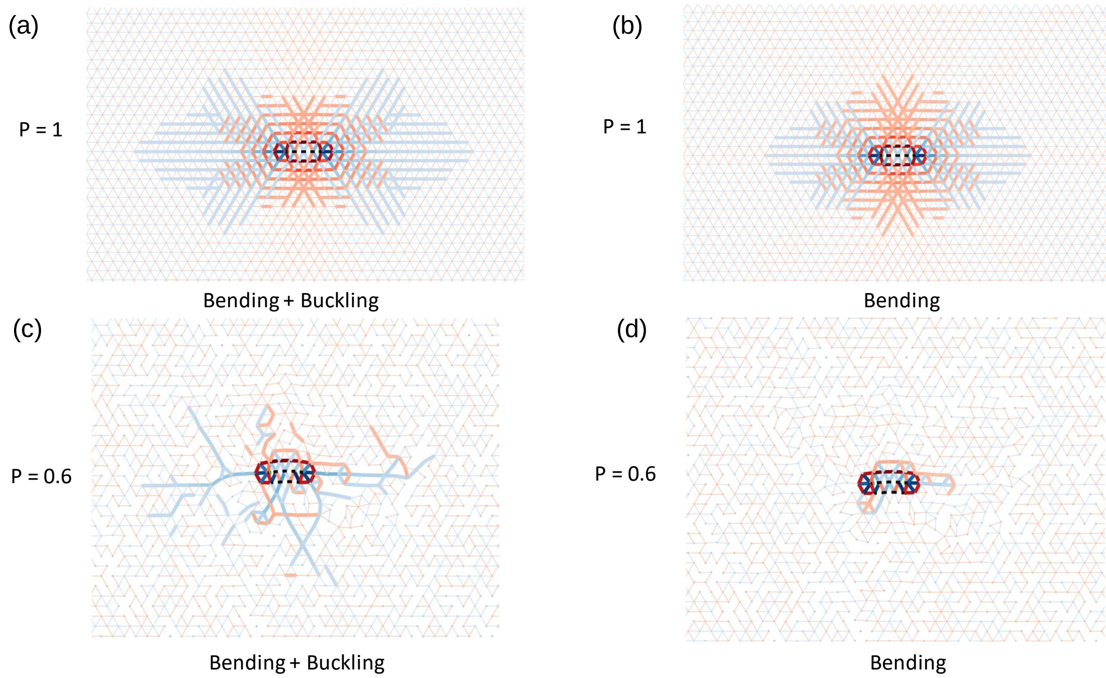

Fig. S11. **Buckling leads to longer force transmission in uniform as well as highly depleted networks. To model buckling, the compression elastic modulus (μ_c) was set to half of tensile elastic modulus (μ). In all of these networks, $\widetilde{\kappa} = 10^{-6}$.** (a) Tensile and compressive force chains with absolute strains more than the threshold value of $|\epsilon_0| > 0.003$ (blue: extensile, red: compressive) in a fully connected network with buckling included. (b) Same as (a) but here the model does not have buckling. (c) Same as (a) but in a depleted network with $p = 0.6$ (d) Same as (c) but the model does not include buckling.