Electronic Supplementary Material (ESI) for Soft Matter.


# Supplemental Information for Mechanical metamaterial inspired by biological tissues

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## I. BAND STRUCTURES FOR VARIOUS STATES

To obtain the full band structure, the dynamical matrix and eigenvalues are calculated along the $\vec{k}$ path $\mathbf{M} = \pi/L(1, 1/\sqrt{3}) \rightarrow \mathbf{\Gamma} = \pi/L(0,0) \rightarrow \mathbf{K} = \pi/L(1/3, 1/\sqrt{3}) \rightarrow \mathbf{M} = \pi/L(1, 1/\sqrt{3})$. $L = \sqrt{N}$ is the box size of the system.

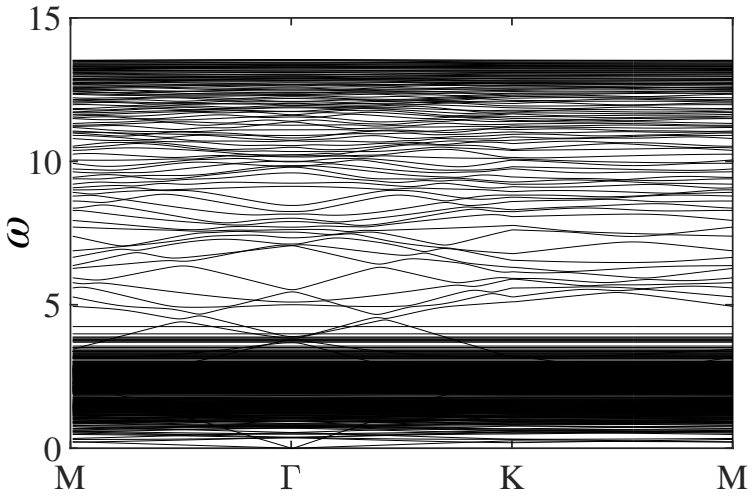


Fig. S1. Band structure for the pure solid state at $N = 100, p_0 = 3.75, \sigma = 0, K_A = 1$. Increasing cell perimeter elasticity $K_P = 10$ does not induce bandgaps.

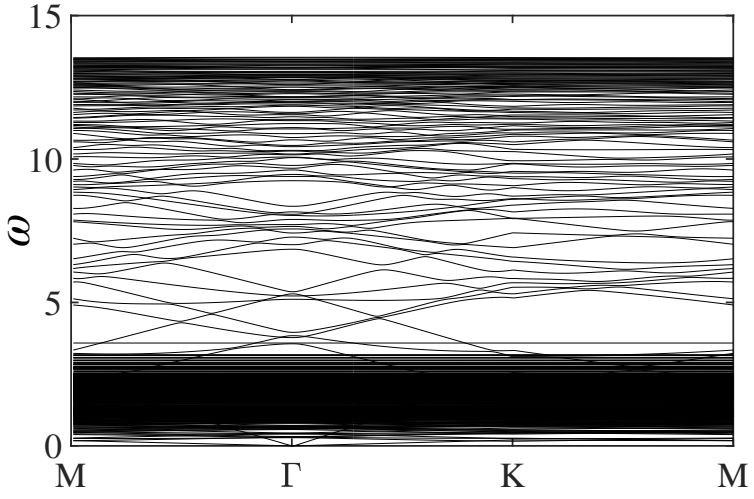


Fig. S2. Band structure for the pure solid state $N = 100, p_0 = 3.75, \sigma = 0.1, K_A = 1$. Increasing cell perimeter elasticity $K_P = 10$ and introducing heterogeneity $\sigma = 0.1$ do not induce bandgaps.

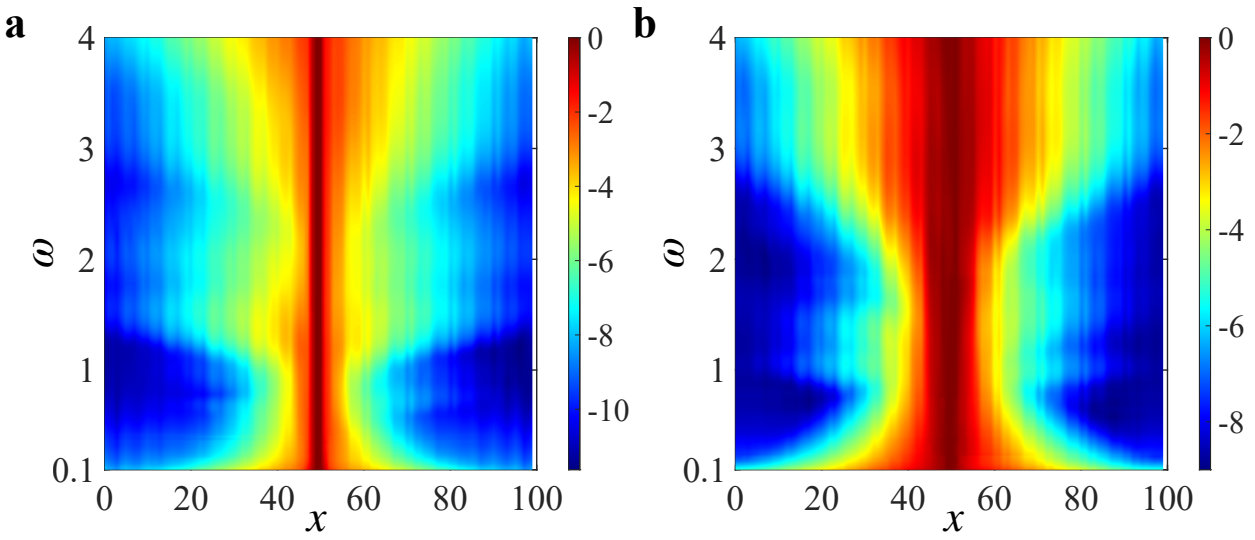


Fig. S3. Colormap of the transmission coefficient $T$ at $N = 100, Nc = 10, p_0 = 3.815, \sigma = 0.1, K_P = 5, K_A = 1$. To reduce the boundary effects, we make $N_c = 10$ copies of the original configuration in the $x$ direction and implement the external sinusoidal perturbation in the (a) transverse and (b) longitude directions. Both of the two colormaps show valleys of $T$ that coincide with the bandgaps indicated by the density of states and band structure in the main text.

## II. DYNAMICAL MATRIX

Energy of a general system $E = T + U$, which consists of kinetic energy $T$ and potential energy $U$. In the harmonic approximation(small oscillation), Taylor expansion of the potential energy

$$U = U_0 + \sum_{i\alpha\mu} H^{\mu}_{i\alpha} + \frac{1}{2}\sum_{i\alpha\mu}\sum_{j\beta\nu} H^{\mu,\nu}_{i\alpha,j\beta} r^{\mu}_{i\alpha} r^{\nu}_{j\beta} + \cdots \tag{1}$$

$H$ is the Hessian matrix given by the second derivative of the energy with respect to vertex positions

$$H^{\mu,\nu}_{i\alpha,j\beta} = \frac{\partial^2 U}{\partial R^{\mu}_{i\alpha} \partial R^{\nu}_{j\beta}}. \tag{2}$$

where $R^{\mu}_{i\alpha}$ is the position of atom $\alpha$ in unit cell $i$ and $\mu = x, y$ are cartesian indices. $H^{\mu}_{i\alpha} = \frac{\partial U}{\partial R^{\mu}_{i\alpha}}$ near the equilibrium position is 0.

Harmonic approximation

$$E = \frac{1}{2}\sum_{i\alpha}^{\mu} M \dot{r}^{\mu}_{i\alpha} \dot{r}^{\mu}_{i\alpha} + \frac{1}{2}\sum_{i\alpha\mu}\sum_{j\beta\nu} H^{\mu,\nu}_{i\alpha,j\beta} r^{\mu}_{i\alpha} r^{\nu}_{j\beta} \tag{3}$$

The equation of motion of the system is

$$M_{\alpha}\ddot{r}^{\mu}_{i\alpha} = -\frac{\partial E}{\partial R^{\mu}_{i\alpha}} = -\sum_{j\beta\nu} H^{\mu,\nu}_{i\alpha,j\beta} r^{\nu}_{j\beta} \tag{4}$$

We can assume periodic solutions get the eigenvalue equations

$$\omega^2 r^{\mu}_{\alpha}(\vec{k}) = \sum_{\beta\nu} D^{\mu,\nu}_{\alpha,\beta}(\vec{k}) r^{\nu}_{\beta}(\vec{k}) \tag{5}$$

Then the dynamical matrix is geven by

$$D^{\mu,\nu}_{\alpha,\beta}(\vec{k}) = \frac{1}{\sqrt{M_{\alpha}M_{\beta}}}\sum_{j} H^{\mu,\nu}_{i\alpha,j\beta} \exp\left[-i\vec{k}\cdot(\vec{R}_i - \vec{R}_j)\right] \tag{6}$$

For an amorphous structure, to calculate the dynamical matrix, the whole system is treated as a super unit cell containing multiple vertices. $\boldsymbol{R}^{\alpha}_{i}$ denotes the position of the unit cell $\alpha$ in which the $i^{th}$ vertex is located. We first take vertex $i$ which is wrapped into the box, then find all its neighboring vertices under periodic boundary condition. In this case, the neighbors might be located outside the initial box. We determine its contribution to the Fourier factor by considering its relative position with respect to the initial box. For example, if the vertex is in the left imaging box, a $-L_x$ translation along $x$ axis, the Fourier factor is $exp(-ik_xL_x)$. If the vertex is in the top right imaging box which is a translation of $L_x$ in $x$ and $L_y$ in $y$ directions, the Fourier factor would be $exp(ik_xL_x + ik_yL_y)$. $L_x, L_y$ are the box sizes of our system in $x, y$ directions, respectively. We use a square box in the simulations $L_x = L_y = L = \sqrt{N}$. $N$ is the number of cells of the tissue.