Supplementary Information: Inertial effect on evasion and pursuit dynamics of prey swarms: the emergence of a favourable mass ratio for the predator-prey arms race

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I. APPENDIX-A

In this section, we analyze the prey-predator dynamics described by Eq. 6 and Eq. 7 in the continuum limit. In our model, for simplicity, we assume that the density distribution of prey swarm is spatially homogeneous. Although the spatial distribution within a swarm can vary, uniform swarm density is a common observation in many swarms, as shown in field studies. For example, Miller and Stephen have found that the flocks of sandhill cranes feeding in cultivated fields exhibit spatial distribution close to uniform, regardless of flock size [S1]. Moroever, tufted ducks or gulls have been found to position themselves evenly within the group according to their body length. There are further instances of nearly uniform distribution of flocks can be seen in nature [S2]. Generally, individuals try to maintain a constant distance between themselves and distribute uniformly. We have modeled the prey-prey interaction by pairwise short-range repulsion and long-range attraction forces. Short-range repulsion helps the group to avoid collisions, and long-range attraction keeps the group cohesive. Uniform swarm density arises due to these opposing forces of attraction and repulsion.

Here, we consider the continuum limit of a large number of prey, which results in the non-local integro-differential equation model [S3]. Thus, in the continuum limit, Eq. 6 can be written as,

$$M_{\rm pr}\frac{d\vec{v}(\vec{x},t)}{dt} + \vec{v}(\vec{x},t) \;=\; \int_{\mathbb{R}^2}[\alpha_0\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2} - \beta_0(\vec{x}-\vec{y})]\rho(\vec{y},t)d\vec{y} + \gamma_0\frac{\vec{x}-\vec{z}}{|\vec{x}-\vec{z}|^2}, \tag{S1}$$

where, $\vec{R}_i$, $\vec{R}_j$, and $\vec{R}_p$ are written in continuum notations by $\vec{x}$, $\vec{y}$, and $\vec{z}$ respectively. $\vec{v}(\vec{x},t) = \frac{d\vec{x}}{dt}$ is the prey velocity at position $\vec{x}$ and time t. We use the fact that in the continuum limit, $\frac{1}{N_{sur}}\sum_{i=1}^{N_{sur}}\delta(\vec{x}-\vec{x_i}) \approx \int_{\mathbb{R}^2}\rho(\vec{y},t)d\vec{y}$ where $\rho(\vec{y},t)$ denotes the prey density distribution which follows

$$\int_{\mathbb{R}^2}\rho_(\vec{y},t)d\vec{y} = 1.$$

Similarly, the equation of motion of the predator (Eq. 7) in the continuum limit can be written as

$$M_{\rm pd}\frac{d^2\vec{z}}{dt^2} + \frac{d\vec{z}}{dt} = \int_{\mathbb{R}^2}\frac{\vec{y}-\vec{z}}{|\vec{y}-\vec{z}|^p}\rho_(\vec{y},t)d\vec{y}. \tag{S2}$$

Now, the continuity equation is:

$$\frac{d\rho}{dt} + \nabla\cdot\rho(\vec{x},t)\vec{v}(\vec{x},t) = 0.$$

Considering ρ is spatially homogeneous *i.e.* $\nabla\rho(\vec{x},t) = 0$; Thus,

$$\nabla\cdot\vec{v}(\vec{x},t) = -\frac{1}{\rho}\frac{d\rho}{dt}. \tag{S3}$$

Taking divergence on both sides of Eq. S1 with respect to $\vec{x}$, we get

$$M_{\rm pr}\Big[\nabla\cdot\frac{d\vec{v}}{dt}\Big] + \nabla\cdot\vec{v} = \rho\int_{\mathbb{R}^2}\Big[\alpha_0\nabla\cdot\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2} - \beta_0\nabla\cdot(\vec{x}-\vec{y})\Big]d\vec{y} + \gamma_0\nabla\cdot\frac{\vec{x}-\vec{z}}{|\vec{x}-\vec{z}|^2} \tag{S4}$$

Using the expression of $(\nabla\cdot\vec{v})$ from Eq. S3 in the above equation, we obtain the following equation:

$$-M_{\rm pr}\frac{d}{dt}(\frac{1}{\rho}\frac{d\rho}{dt}) - \frac{1}{\rho}\frac{d\rho}{dt} = 2\pi\gamma_0\delta(\vec{x}-\vec{z}) + \rho\int_{\mathbb{R}^2}\Big[(2\pi\alpha_0\delta(\vec{x}-\vec{y}) - 2\beta_0)d\vec{y}\Big], \tag{S5}$$

where we have used the following identities:

$$\nabla\cdot\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2} = 2\pi\delta(\vec{x}-\vec{y}),$$

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$$\nabla \cdot (\vec{x}-\vec{y}) = 2.$$

Since, $\delta(\vec{x}-\vec{z}) = 0$ (as $\vec{x} \neq \vec{z}$), we can write Eq. S5 as

$$-M_{\rm pr}\frac{d}{dt}(\frac{1}{\rho}\frac{d\rho}{dt}) - \frac{1}{\rho}\frac{d\rho}{dt} = 2\pi\alpha_0\rho - 2\beta_0,$$

$$M_{\rm pr}[\frac{1}{\rho}\frac{d^2\rho}{dt^2} - \frac{1}{\rho^2}(\frac{d\rho}{dt})^2] + \frac{1}{\rho}\frac{d\rho}{dt} = 2\beta_0 - 2\pi\alpha_0\rho.$$

Introducing a new variable $w = \frac{d\rho}{dt}$, the last equation becomes

$$M_{\rm pr}[\frac{1}{\rho}\frac{dw}{dt} - \frac{1}{\rho^2}w^2] + \frac{w}{\rho} = 2\beta_0 - 2\pi\alpha_0\rho. \quad \text{(S6)}$$

In the steady state, $\frac{d\rho}{dt} = 0$ & $\frac{dw}{dt} = 0$. This gives $w^s = 0$, and $\rho^s = \frac{\beta_0}{\pi\alpha_0}$, where w^s and ρ^s are the respective steady state values of w and ρ. It turns out that ρ^s is independent of predator and prey mass.

Now, in the case of a weak predator, from the simulations, we find that in the steady state, the predator stays at the center and prey group circles around the predator, as shown in Fig. 2a. Further, we analytically calculate the inner radius (R_1) and outer radius (R_2) of the annulus (A) formed by the prey group at the steady state, as shown in Fig. S1. Thus, we can write,

$$M_{\rm pr}\frac{d\vec{v}}{dt} + \vec{v} \;=\; \rho^s \int_{R_1}^{R_2} (\alpha_0\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2} - \beta_0(\vec{x}-\vec{y}))d\vec{y} + \gamma_0\frac{\vec{x}-\vec{z}}{|\vec{x}-\vec{z}|^2}. \quad \text{(S7)}$$

Using the identities $\int_{|\vec{y}|\leq R}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y} = \pi\vec{x}$ for $|\vec{x}| < R$, and $\int_{|\vec{y}|\leq R}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y} = \pi R^2\frac{\vec{x}}{|\vec{x}|.^2}$ for $|\vec{x}| > R$, we get $\int_0^{R_2}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y} = \pi\vec{x}$, and $\int_0^{R_1}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y} = \pi R_1^2\frac{\vec{x}}{|\vec{x}|.^2}$. Considering polar symmetry, it can be shown that $\int_{R_1}^{R_2}\vec{y}d\vec{y} = 0$. Thus, Eq. S7 reduces to the following form:

$$M_{\rm pr}\frac{d\vec{v}}{dt} + \vec{v} \;=\; \pi\rho^s\vec{x}[\alpha_0 - \beta_0(R_2^2 - R_1^2)] - \pi\rho^s\alpha_0 R_1^2\frac{\vec{x}}{|\vec{x}|^2} + \gamma_0\frac{\vec{x}-\vec{z}}{|\vec{x}-\vec{z}|^2}, \quad \text{(S8)}$$

where we have used $\int_0^R d\vec{y} = \pi R^2$. At the steady state $\frac{d\vec{x}}{dt} = 0$ and $\frac{d\vec{v}}{dt} = 0$; let $\vec{x} = \vec{x^*}$ and $\vec{v} = \vec{v^*}$ be the respective steady state values of $\vec{x}$ and $\vec{v}$. Then, from Eq. S8, we get

$$\pi\rho\vec{x^*}[\alpha_0 - \beta_0(R_2^2 - R_1^2)] + \gamma_0\frac{\vec{x^*}-\vec{z^*}}{|\vec{x^*}-\vec{z^*}|^2} - \pi\rho\alpha_0 R_1^2\frac{\vec{x}}{|\vec{x}|^2} = 0,$$

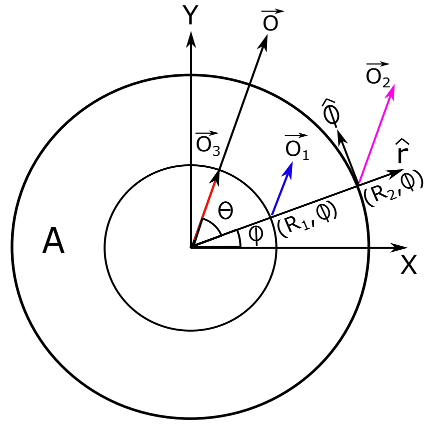


FIG. S1. $\vec{O}$ is the general perturbation and $\vec{O_1}$, $\vec{O_2}$, $\vec{O_3}$ are its values at the inner boundary, outer boundary, and predator position. 'A' is specified as the annular region, as shown in the diagram.

where $\vec{z^*}$ is the steady state value of $\vec{z}$. By assuming that the predator sits at the origin of the annulus at the steady state, we finally arrive at the following equation:

$$\pi\rho\vec{x^*}[\alpha_0 - \beta_0(R_2^2 - R_1^2)] + (\gamma_0 - \pi\rho\alpha_0 R_1^2)\frac{\vec{x^*}}{|\vec{x^*}|^2} = 0.$$

For a non-trivial solution, $\alpha_0 - \beta_0(R_2^2 - R_1^2) = 0$ and $\gamma_0 - \alpha_0\pi\rho R_1^2 = 0$. Hence,

$$R_1 = \sqrt{\frac{\gamma_0}{\beta_0}},$$

$$R_2 = \sqrt{\frac{\alpha_0 + \gamma_0}{\beta_0}}.$$

So, we can infer that the inner(R_1) and outer(R_2) radii of the ring depend on the prey-prey and prey-predator interaction strengths α_0, β_0, γ_0 but are independent of prey and predator mass.

II. APPENDIX-B

In this section, we analyze the stability criteria for the ring (A). For that we perturb the ring by a general perturbation vector $\vec{o}$ with an angle θ with respect to the origin and $\hat{r}$ as the direction of the perturbation, such that $\vec{o} = \vec{o_1}$ at the inner boundary, $\vec{o} = \vec{o_2}$ on the outer boundary and $\vec{o} = \vec{o_3}$ at the position of the predator as depicted in the Fig. S1. The direction of the perturbation always stays constant along the direction $\hat{r}$. With perturbation $\vec{o}$, the Eq. S7 becomes

$$M_{\rm pr}\frac{d\vec{v}}{dt} + \vec{v} \;=\; \rho\int_{R_1}^{R_2}\Big[\alpha_0\frac{\vec{x}-(\vec{y}+\vec{o})}{|\vec{x}-(\vec{y}+\vec{o})|^2} - \beta_0(\vec{x}-(\vec{y}+\vec{o})\Big]d\vec{y} + \gamma_0\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}, \quad \text{(S9)}$$

$$M_{\rm pr}\frac{d\vec{v}}{dt}+\vec{v}=\rho\int_{R_1}^{R_2}\Big[\alpha_0\frac{\vec{u}-\vec{y}}{|\vec{u}-\vec{y}|^2}-\beta_0(\vec{u}-\vec{y})\Big]d\vec{y}+\gamma_0\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2},$$

where $\vec{u}=\vec{x}-\vec{o}$, such that $\vec{u}=\vec{u_2}=\vec{x}-\vec{o_2}$ on the outer boundary, and $\vec{u}=\vec{u_1}=\vec{x}-\vec{o_1}$ on the inner boundary. Using the previous results $\int_0^{R_2}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y}=\pi\vec{x}$, $\int_0^{R_1}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}|^2}d\vec{y}=\pi R_1^2\frac{\vec{x}}{|\vec{x}|.^2}$ and $\int_{R_1}^{R_2}\vec{y}d\vec{y}=0$, we get,

$$M_{\rm pr}\frac{d\vec{v}}{dt}+\vec{v}=\rho\Big[\pi\alpha_0\vec{u_2}-\pi\alpha_0R_1^2\frac{\vec{u_1}}{|\vec{u_1}|^2}-\beta_0(\vec{u_2}\pi R_2^2-\vec{u_1}\pi R_1^2)\Big]+\gamma_0\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}. \quad \text{(S10)}$$

Substituting $\vec{u_1}$ and $\vec{u_2}$ in the above equation, we obtain

$$M_{\rm pr}\frac{d\vec{v}}{dt}+\vec{v}=\gamma_0\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}+\rho\Big[(\pi\vec{x}-\pi\vec{o_2})\alpha_0+\beta_0\pi\vec{x}(R_1^2-R_2^2)-\pi\alpha_0R_1^2\frac{\vec{x}-\vec{o_1}}{|\vec{x}-\vec{o_1}|^2}+\beta_0\pi(R_2^2\vec{o_2}-R_1^2\vec{o_1})\Big]. \quad \text{(S11)}$$

At the steady state, $\vec{o}_i=0$, $\vec{v}=0$, $\frac{d\vec{v}}{dt}=0$ and $\rho^s=\frac{\beta_0}{\alpha_0\pi}$ as stated above. Using these conditions in the Eq. S11, we get,

$$\pi\alpha_0\rho+\pi\rho\beta_0(R_1^2-R_2^2)=0. \quad \text{(S12)}$$

Combining Eq. S11 and Eq. S12, we finally arrive at the following equation:

$$M_{\rm pr}\frac{d\vec{v}}{dt}+\vec{v}=\rho\pi[(\beta_0R_2^2-\alpha_0)\vec{o_2}-\beta_0R_1^2\vec{o_1}]+\rho\pi\alpha_0R_1^2\Big[\frac{\vec{x}}{|\vec{x}|^2}-\frac{\vec{x}-\vec{o_1}}{|\vec{x}-\vec{o_1}|^2}\Big]+\gamma_0\Big[\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}-\frac{\vec{x}}{|\vec{x}|^2}\Big]. \quad \text{(S13)}$$

Next, we linearize the Eq. S13 on the inner boundary, outer boundary, and at the predator position. On the inner boundary,

$$\vec{x}=R_1\hat{r}+\vec{o_1}(\hat{r},\hat{\theta}).$$

Hence, on the inner boundary, we can show

$$\frac{\vec{x}}{|\vec{x}|^2}-\frac{\vec{x}-\vec{o_1}}{|\vec{x}-\vec{o_1}|^2}=\frac{1}{R_1^2}[\vec{o_1}-2o_1cos\theta\hat{r}], \quad \text{(S14)}$$

and

$$\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}-\frac{\vec{x}}{|\vec{x}|^2}=-\frac{1}{R_1^2}[\vec{o_3}-2o_3cos\theta\hat{r}], \quad \text{(S15)}$$

where the terms other than the linear order are neglected due to smallness. Using Eq. S14 and Eq. S15, the Eq. S13 can be written as:

$$M_{\rm pr}\frac{d\vec{v}}{dt}+\vec{v}=\rho\pi((\beta_0R_2^2-\alpha_0)\vec{o_2}-\beta_0R_1^2\vec{o_1})+\rho\pi\alpha_0(\vec{o_1}-2o_1cos\theta\hat{r})-\frac{\gamma_0}{R_1^2}[\vec{o_3}-2o_3cos\theta\hat{r}]. \quad \text{(S16)}$$

Since, $\hat{r}$ is taken as a constant, $\vec{v}=\frac{d\vec{o_1}}{dt}$ and $\frac{d\vec{v}}{dt}=\frac{d^2\vec{o_1}}{dt^2}$ on the inner boundary. Hence, the Eq. S16 becomes

$$M_{\rm pr}\frac{d^2\vec{o_1}}{dt^2}+\frac{d\vec{o_1}}{dt}=\rho\pi((\beta_0R_2^2-\alpha_0)\vec{o_2}-\beta_0R_1^2\vec{o_1})+\rho\pi\alpha_0(\vec{o_1}-2o_1cos\theta\hat{r})-\frac{\gamma_0}{R_1^2}[\vec{o_3}-2o_3cos\theta\hat{r}]. \quad \text{(S17)}$$

Taking scalar product on both sides of Eq. S17 by $\hat{r}$, the Eq. S17 reduces to the following form:

$$M_{\rm pr}\frac{d^2o_1}{dt^2}+\frac{do_1}{dt}=\rho\pi(-\beta_0R_1^2-\alpha_0)o_1+\rho\pi(\beta_0R_2^2-\alpha_0)o_2+\frac{\gamma_0o_3}{R_1^2}. \quad \text{(S18)}$$

In terms of the system parameters, the last equation is

$$M_{\rm pr}\frac{d^2o_1}{dt^2}+\frac{do_1}{dt}=-\frac{\beta_0}{\alpha_0}(\gamma_0+\alpha_0)o_1+\frac{\beta_0\gamma_0}{\alpha_0}o_2+\beta_0o_3. \quad \text{(S19)}$$

Next, we linearize the Eq. S13 on the outer boundary where $\vec{x}=R_2\hat{r}+\vec{o_2}$. Using similar calculations as done previously, we get the following equations:

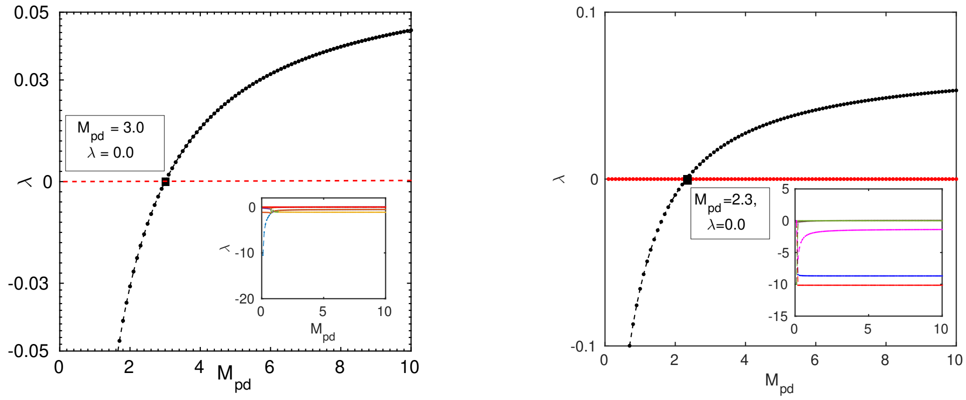
$$\frac{\vec{x}}{|\vec{x}|^2}-\frac{\vec{x}-\vec{o_1}}{|\vec{x}-\vec{o_1}|^2}=\frac{1}{R_2^2}[\vec{o_1}-2o_1cos\theta\hat{r}], \quad \text{(S20)}$$

$$\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^2}-\frac{\vec{x}}{|\vec{x}|^2}=-\frac{1}{R_2^2}[\vec{o_3}-2o_3cos\theta\hat{r}], \quad \text{(S21)}$$

$$M_{\rm pr}\frac{d^2o_2}{dt^2}+\frac{do_2}{dt}=-\frac{\beta_0\gamma_0}{\alpha_0}(1+\frac{\alpha_0}{\alpha_0+\gamma_0})o_1+\frac{\beta_0\gamma_0}{\alpha_0}o_2+\frac{\beta_0\gamma_0}{\alpha_0+\gamma_0}o_3. \quad \text{(S22)}$$

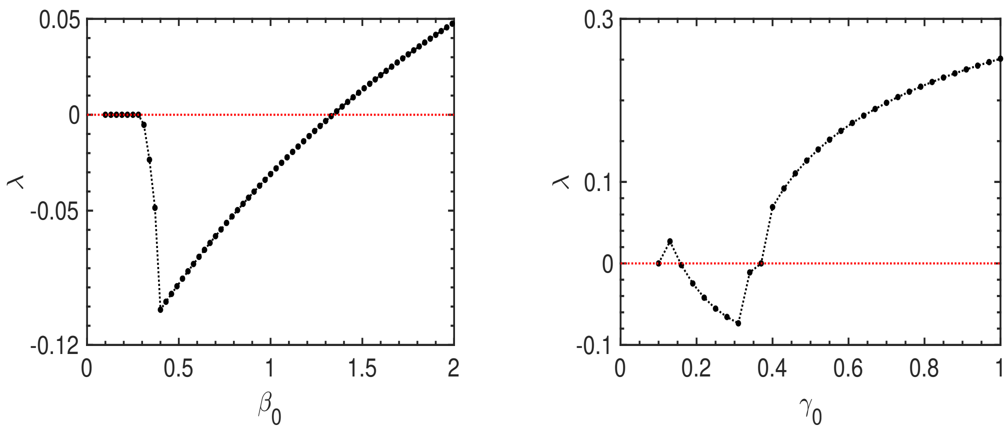
We next linearize the predator equation (Eq. S2). Since $\vec{o_3}$ is the perturbation at the position of the predator about its steady state(origin), therefore $\vec{z}=\vec{o_3}$.
Hence, with perturbation, the Eq. S2 becomes

$$M_{\rm pd}\frac{d^2\vec{o_3}}{dt^2}+\frac{d\vec{o_3}}{dt}=\delta_0\rho\int_{B(o_2,R_2)/(o_1,R_1)}\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^p}d\vec{x}, \quad \text{(S23)}$$



(a) (b)

FIG. S2. Eigenvalue has been plotted as a function of predator mass M_{pd} in (a) and (b) where the prey mass M_{pr} is kept constant to 1.0 and 0.1 respectively. Eigenvalue crosses zero at $M_{\mathrm{pd}} = 3.0$ and $M_{\mathrm{pd}} = 2.3$ in the case of (a) and (b), respectively. Other eigenvalues of the transition matrix are negative, as shown in the insets.



(a) (b)

FIG. S3. One of the eigenvalues of the transition matrix has been plotted with varying (a) β_0 and (b) γ_0, while keeping all other parameters constant. In scenario (a), the parameters are set as: $M_{\mathrm{pr}} = 1.0$, $M_{\mathrm{pd}} = 2.0$, $\gamma_0 = 0.2$, $\alpha_0 = 1.0$, and $\delta_0 = 0.4$. However, in the case of (b), the parameters remain the same as those in (a), except for the variable γ_0, which is altered while keeping the value of $\beta_0 = 1$.

where $B(o_2, R_2)/(o_1, R_1)$ denotes the annular region A with perturbations o_1 and o_2 on its inner and outer boundaries respectively. Now, $B(o_2, R_2)/(o_1, R_1)$ and A follows the following transformation relation:

$$\int_{B(o_2,R_2)/(o_1,R_1)} \frac{\vec{x}}{x^p} d\vec{x} = \int_A \frac{\vec{x}+\vec{o}}{|\vec{x}+\vec{o}|^p} d\vec{x}. \tag{S24}$$

Assuming $|\vec{o_3}|^2$ is small, we can show that

$$\frac{\vec{x}-\vec{o_3}}{|\vec{x}-\vec{o_3}|^p} \approx \frac{\vec{x}}{x^p} + \frac{p\vec{x}(\vec{x}\cdot\vec{o_3}) - \vec{o_3}|\vec{x}|^2}{x^{p+2}}.$$

So, the Eq. S23 becomes

$$M_{\mathrm{pd}} \frac{d^2\vec{o_3}}{dt^2} + \frac{d\vec{o_3}}{dt} \approx \delta_0 \rho \int_{B(o_2,R_2)/(o_1,R_1)} \left[\frac{\vec{x}}{x^p} + \frac{p\vec{x}(\vec{x}\cdot\vec{o_3}) - \vec{o_3}|\vec{x}|^2}{x^{p+2}}\right] d\vec{x}. \tag{S25}$$

Using the transformation relation Eq. S24, one can show that

$$\int_{B(o_2,R_2)/(o_1,R_1)} \frac{\vec{x}}{x^p} d\vec{x} \approx \int_A \frac{\vec{x}}{x^p} d\vec{x} + \vec{o} \int_A \frac{d\vec{x}}{x^p} - p \int_A \frac{\vec{x}(\vec{x}\cdot\vec{o})}{x^{p+2}} d\vec{x}. \tag{S26}$$

Using $\vec{x} = x[\hat{i}\cos\theta + \hat{j}\sin\theta]$, and $d\vec{x} = x dx d\theta$, we can find the following integration results:

$$\int_A \frac{\vec{x}}{x^p} d\vec{x} = 0,$$

$$\int_A \frac{\vec{o}}{x^p} d\vec{x} = \frac{2\pi}{2-p}[\vec{o_2} R_2^{2-p} - \vec{o_1} R_1^{2-p}],$$

$$\int_A \frac{\vec{x}(\vec{x}\cdot\vec{o})}{x^{p+2}} d\vec{x} = \frac{\pi R_2^{2-p}\vec{o_2} - \pi R_1^{2-p}\vec{o_1}}{2-p}.$$

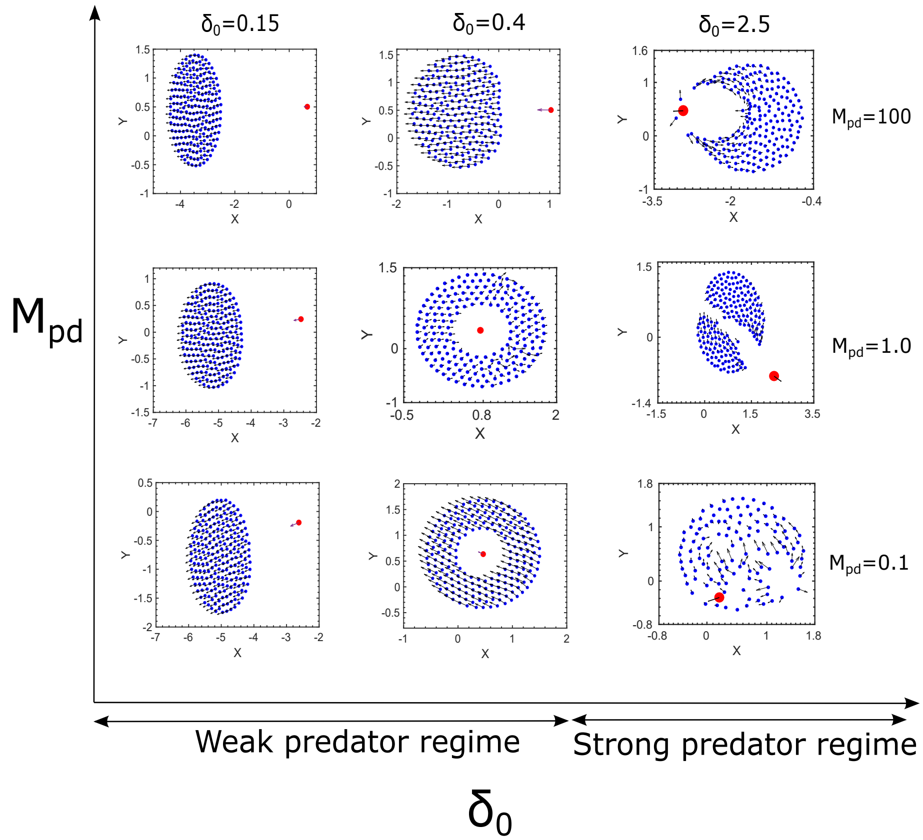


FIG. S4. An overview of various escape trajectories of the prey swarm that emerge with increasing predator strength, δ_0, and predator mass, $M_{\rm pd}$, while keeping the prey mass constant at $M_{\rm pr} = 1.0$.

So, the Eq. S26 becomes

$$\int_{B(o_2,R_2)/(o_1,R_1)} \frac{\vec{x}}{x^p} d\vec{x} \approx \pi(R_2^{2-p}\vec{o_2} - R_1^{2-p}\vec{o_1}) \tag{S27}$$

Next, we need to integrate the following integration:

$$\int_{B(o_2,R_2)/(o_1,R_1)} \frac{p\vec{x}(\vec{x}\cdot\vec{o_3}) - \vec{o_3}|\vec{x}|^2}{x^{p+2}} d\vec{x}$$

$$= p\int_{B(o_2,R_2)/(o_1,R_1)} \frac{\vec{x}(\vec{x}\cdot\vec{o_3})}{|\vec{x}|^{p+2}} d\vec{x} - \vec{o_3}\int_{B(o_2,R_2)/(o_1,R_1)} \frac{d\vec{x}}{x^p} \tag{S28}$$

$= I_1 + I_2$,
where $I_1 = p\int_{B(o_2,R_2)/(o_1,R_1)} \frac{\vec{x}(\vec{x}\cdot\vec{o_3})}{|\vec{x}|^{p+2}} d\vec{x}$,
and
$I_2 = -\vec{o_3}\int_{B(o_2,R_2)/(o_1,R_1)} \frac{d\vec{x}}{x^p}$.
Using the transformation relation Eq. S24, we can write

$$I_1 = p\int_A \frac{(\vec{x}+\vec{o})((\vec{x}+\vec{o})\cdot\vec{o_3})}{|\vec{x}+\vec{o}|^{p+2}} d\vec{x}.$$

Considering only the linear order terms in $\vec{o}$ and $\vec{o_3}$, we can show

$$I_1 = \frac{\pi p\vec{o_3}}{2-p}(R_2^{2-p} - R_1^{2-p}). \tag{S29}$$

Similarly, we can show

$$I_2 = -\frac{2\pi\vec{o_3}}{2-p}(R_2^{2-p} - R_1^{2-p}). \tag{S30}$$

Putting the expressions of I_1 and I_2 from Eq. S29 and Eq. S30, finally Eq. II becomes

$$\int_{B(o_2,R_2)/(o_1,R_1)} \frac{p\vec{x}(\vec{x}\cdot\vec{o_3}) - \vec{o_3}|\vec{x}|^2}{x^{p+2}} d\vec{x} = -\pi\vec{o_3}(R_2^{2-p} - R_1^{2-p}). \tag{S31}$$

Now the Eq. S26 can be written as:

$$M_{\rm pd}\frac{d^2\vec{o_3}}{dt^2} + \frac{d\vec{o_3}}{dt} \approx \rho\delta_0[-\pi R_1^{2-p}\vec{o_1} + \pi R_2^{2-p}\vec{o_2} + \pi\vec{o_3}(R_1^{2-p} - R_2^{2-p})].$$

In terms of the system parameters, the last equation becomes

$$M_{\rm pd}\frac{d^2\vec{o_3}}{dt^2} + \frac{d\vec{o_3}}{dt} = \frac{\delta_0\beta_0}{\alpha_0}\Big[-(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}}\vec{o_1} + (\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}}\vec{o_2} + [(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}} - (\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}}]\vec{o_3}\Big] \tag{S32}$$

Since $\vec{o_1}$, $\vec{o_2}$, $\vec{o_3}$ are aligned along the same direction, therefore, linearized predator equation will be

$$M_{\rm pd}\frac{d^2o_3}{dt^2}+\frac{do_3}{dt} = \frac{\delta_0\beta_0}{\alpha_0}\Big[-(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}}o_1+(\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}}o_2 + [(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}}-(\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}}]o_3\Big] \quad (S33)$$

Introducing a set of new variables $\vec{u}_i = \frac{d\vec{o}_i}{dt}$, where $i = 1, 2, 3$. Finally, we get the Jacobian matrix as shown in S35. Eq. S19, Eq. S22, and Eq. S33 can be written in matrix form as shown in Eq. S34. The dynamics of the system could be predicated by analyzing the eigenvalues of the transition matrix M. Numerically, the steady state structure has been observed at $\delta_0 = 0.4, \beta_0 = 1.0, \gamma_0 = 0.2, \alpha_0 = 1.0$, $p = 3$ for smaller prey and predator masses. Hence, we put similar parameters in the Jacobian matrix M and then investigate how the eigenvalues behave as we vary the prey and predator masses. Fig. 2(a) clearly demonstrates the evolution of eigenvalues as we change the $M_{\rm pd}$; one of the eigenvalues goes from positive to negative at $M_{\rm pd} = 3.0$ whereas other eigenvalues remain negative. Further, we have also investigated the eigenvalues for another mass of prey, $M_{\rm pr} = 0.1$, as depicted in Fig. 2(b); here also one of the eigenvalues shows the transition from negative to positive values at a predator mass, $M_{\rm pd} = 2.3$. Thus, it is evident from the eigenvalue analysis that the stable ring becomes unstable at a particular predator mass, keeping the prey mass constant. Hence, we could infer that inertia of both prey and the predator have a profound effect on the prey-predator dynamics. It is important to note that the stability of the ring not only depends on the value of $M_{\rm pr}$ and $M_{\rm pd}$, but also on other parameter values such as β_0, γ_0, and α_0 as they are integral components of the transition matrix (M). We have presented the variation of one of the eigenvalues of the transition matrix (M) by varying β_0 and γ_0 in Fig. S3, keeping other parameters constant as stated in the caption of the figure. It is evident from Fig. S3 that the stable ring formed by the prey swarm becomes unstable (*i.e.*, the value of the eigenvalue crosses zero) with increasing β_0 and γ_0 values. Moreover, Fig. S4 provides an overview of various escape routes of the prey swarm that emerge with increasing predator mass and predator strength while keeping the prey mass constant, as discussed in the paper.

$$\begin{pmatrix}\frac{d\vec{o_1}}{dt}\\ \frac{d\vec{o_2}}{dt}\\ \frac{d\vec{o_3}}{dt}\\ \frac{d\vec{u_1}}{dt}\\ \frac{d\vec{u_2}}{dt}\\ \frac{d\vec{u_3}}{dt}\end{pmatrix} = M\begin{pmatrix}\vec{o_1}\\ \vec{o_2}\\ \vec{o_3}\\ \vec{u_1}\\ \vec{u_2}\\ \vec{u_3},\end{pmatrix} \quad (S34)$$

$$M = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ -\frac{\beta_0}{\alpha_0 M_{\rm pr}}(\gamma_0+\alpha_0) & \frac{\beta_0\gamma_0}{\alpha_0 M_{\rm pr}} & \frac{\beta_0}{M_{\rm pr}} & -\frac{1}{M_{\rm pr}} & 0 & 0\\ -\frac{\beta_0\gamma_0}{\alpha_0 M_{\rm pr}}(1+\frac{\alpha_0}{\alpha_0+\gamma_0}) & \frac{\beta_0\gamma_0}{\alpha_0 M_{\rm pr}} & \frac{\beta_0\gamma_0}{M_{\rm pr}(\alpha_0+\gamma_0)} & 0 & -\frac{1}{M_{\rm pr}} & 0\\ -\frac{\delta_0\beta_0}{\alpha_0 M_{\rm pd}}(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}} & \frac{\delta_0\beta_0}{\alpha_0 M_{\rm pd}}(\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}} & \frac{\delta_0\beta_0}{\alpha_0 M_{\rm pd}}[(\frac{\gamma_0}{\beta_0})^{\frac{2-p}{2}}-(\frac{\alpha_0+\gamma_0}{\beta_0})^{\frac{2-p}{2}}] & 0 & 0 & -\frac{1}{M_{\rm pd}} \end{pmatrix} \quad (S35)$$

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