

Electronic Supplementary Information for: Theoretical Treatment of Complex Coacervate Core Micelles: Structure and pH-Induced Disassembly

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Extremization of the semi-grand canonical free energy functional

We find the extreme the Lagrangian functional given by the semi-grand canonical free energy of the system and the constraints enforced through the use of Lagrange multipliers (Eq. 20 in the main text) with respect to the following functions: $\rho_i(r)$, $P_i(\alpha_i, r)$ and $f_j^k(r)$. The extreme with respect to the density distribution of solvent results in the expression,

$$\rho_s(r) = \rho_s^{bulk} \exp[-\beta v_s (\pi(r) - \pi^{bulk})] \quad (\text{S1})$$

where ρ_s^{bulk} is the density of the solvent in the bulk.¹

The extreme with respect to respect the density distribution of the mobile species $i = \text{H}^+, \text{OH}^-, \text{K}^+, \text{Cl}^-$, results in:

$$\rho_i(r) = \rho_i^{bulk} \exp\{-\beta q_i [\psi(r) - \psi^{bulk}] - \beta v_i [\pi(r) - \pi^{bulk}]\} \quad (\text{S2})$$

The bulk potential, ψ^{bulk} , is set to zero hereafter. For the probability distribution function of molecular conformations $P_i(\alpha_i, r)$ the final expression is:

$$P_{copol}(\alpha_{copol}, r) = Q_{copol}(r)^{-1} \exp\left[-\sum_{k=A,N} \int n_{copol,k}(r', \alpha_{copol}, r) v_p \beta \pi(r') dr' - \int n_{copol,A}(r', \alpha_{copol}, r) \left(\ln(f_c^A(r')) + \beta \psi(r') q_{A-}\right) dr'\right] \quad (\text{S3})$$

$$P_{guest}(\alpha_{guest}, r) = Q_{guest}(r)^{-1} \exp\left[-\int n_{guest,B}(r', \alpha_{guest}, r) \left(\ln(f_c^B(r')) + \beta \psi(r') q_{B-} + v_p \beta \pi(r')\right) dr'\right] \quad (\text{S4})$$

where $Q_i(r) = \exp(1 + \xi_i(r)/[G(r)\rho_i(r)])$.

The minimization with respect to the fraction of segments in different states, f_j^A for copolymer A segments (with $j = c, uc, A-B, A-ion$) and f_j^B for guest B segments (with $j = c, uc, A-B, B-ion$) results in the expressions of the acid-base equilibrium and the copolymer/guest, salt-ion/copolymer and salt-ion/guest ion-pairing equilibria.

The acid-base equilibrium of the A groups in the copolymer (Eq. 2 in the main text) is given by:

$$K_A^\theta = \frac{f_c^A(r)\rho_{H^+}(r)}{f_{uc}^A(r)\rho_s(r)} \quad (S5)$$

where $K_A^\theta = \exp(-\beta\Delta G_A^\theta) = \exp[-\beta(\mu_{H^+}^\theta + \mu_c^{\theta,A} - \mu_{uc}^{\theta,A})]$ is the thermodynamic equilibrium constant of the acid-base reaction.

In analogy, the acid-base equilibrium of the base groups of the guest molecule (8) is,

$$K_B^\theta = \frac{f_c^B(r)\rho_{OH^-}(r)}{f_{uc}^B(r)\rho_s(r)} \quad (S6)$$

where $K_B^\theta = \exp(-\beta\Delta G_B^\theta) = \exp[-\beta(\mu_{OH^-}^\theta + \mu_c^{\theta,B} - \mu_{uc}^{\theta,B})]$.

The thermodynamic constants K_A^θ and K_B^θ are related to the equilibrium constants in molar reference states by dividing them by a factor $c_0 N_A v_s / (10^{24} \text{ nm}^3 / \text{dm}^3)$, where c_0 is the standard concentration (1 M) and N_A is Avogadro's number.

The equilibrium reaction for ion pairing between copolymer A segments and salt cations (Eq. 4 in the main text) is given by:

$$K_{A\text{-ion}}^\theta = \frac{f_{A\text{-ion}}^A(r)(\rho_s(r)v_s)^{v_{K^+}/v_s}}{f_c^A(r)\rho_{K^+}(r)v_s} \quad (S7)$$

where $K_{A\text{-ion}}^\theta = \exp[\beta(\mu_{K^+}^\theta + \mu_c^{\theta,A} - \mu_{A\text{-ion}}^{\theta,A})]$. The equation for ion pairing between anions and B segments in the guest molecule (Eq. 5 in the main text) is:

$$K_{B\text{-ion}}^\theta = \frac{f_{B\text{-ion}}^B(r)(\rho_s(r)v_s)^{v_{Cl^-}/v_s}}{f_c^B(r)\rho_{Cl^-}(r)v_s} \quad (S8)$$

where $K_{B\text{-ion}}^\theta = \exp[\beta(\mu_{Cl^-}^\theta + \mu_c^{\theta,B} - \mu_{B\text{-ion}}^{\theta,B})]$.

In the dilute limit ($\rho_s^{bulk} v_s \rightarrow 1$), the thermodynamic constants $K_{A\text{-ion}}^\theta$ and $K_{B\text{-ion}}^\theta$ can be transformed to association constants using molar concentrations by multiplication by $c_0 N_A v_s / (10^{24} \text{ nm}^3 / \text{dm}^3)$.

The association equilibrium between copolymer and guest segments (Eq. 1 in the main text) is

$$K_{\text{A-B}}^\theta v_{\text{AB}} = \frac{f_{\text{A-B}}^{\text{A}}(r)}{f_c^{\text{A}}(r)f_c^{\text{B}}(r)\langle n_{\text{B}}(r) \rangle} = \frac{f_{\text{A-B}}^{\text{B}}(r)}{f_c^{\text{A}}(r)f_c^{\text{B}}(r)\langle n_{\text{A}}(r) \rangle} \quad (\text{S9})$$

where $K_{\text{A-B}}^\theta = \exp \left[\beta(\mu_{\text{A-B}}^{\theta,\text{A}} + \mu_{\text{A-B}}^{\theta,\text{B}} - \mu_c^{\theta,\text{A}} - \mu_c^{\theta,\text{B}}) \right]$

Once again, it is possible to obtain the association constant in the molar reference state, $K_{\text{A-B}}$, as $K_{\text{A-B}} = K_{\text{A-B}}^\theta v_{\text{AB}} c_0 N_{\text{A}} / (10^{24} \text{ nm}^3 / \text{dm}^3)$.

The functional extreme respect to the electrostatic potential, $\psi(r)$, results in

$$\nabla \cdot (\epsilon \nabla \psi(r)) = -\langle \rho_q(r) \rangle \quad (\text{S10})$$

Eq. S10 is a mean-field Poisson equation for electrostatics, where the electrostatic potential and the number density of charges are replaced by their ensemble averages. The boundary conditions for this equation are $\psi(r \rightarrow \infty) = 0$ (bulk) and symmetric conditions (null derivative) at $r = 0$:

$$\left. \frac{\partial \psi(r)}{\partial r} \right|_{r=0} = 0 \quad (\text{S11})$$

Additional information on the derivation and solving of the molecular theory (MOLT) can be found in our previous publication of amphiphile assemblies²⁻⁴ and polyelectrolyte complexes.^{5,6}

Discretization and numerical solving

The set of Eqs. S1-S10 is solved by discretizing these equations and the packing constraint (Eq. 16 in the main text) in a grid along the r -direction, using layers of thickness $\delta = 0.2$ nm. The set of resulting non-linear equations is solved numerically using a Jacobian-free Newton method.⁷

Calculation of the radii of the core and the micelle from the volume-fraction profiles of the segments

We use the volume-fraction profiles of each type of segment $\langle\phi_A(r)\rangle$, $\langle\phi_B(r)\rangle$ and $\langle\phi_N(r)\rangle$ to calculate the core and total radius of the C3Ms from MOLT. In the case of core radius R_{core} , we first sum the volume fractions of the charged segments to obtain the volume fraction profile of the core $\langle\phi_{core}(r)\rangle = \langle\phi_A(r)\rangle + \langle\phi_B(r)\rangle$. We then obtain R_{core} from the equimolar dividing surface using a definition of that is typically used in molecular dynamic simulations to determine the radii of droplets.⁸ More specifically, R_{core} is given by:

$$R_{core} = \left[\frac{1}{\langle\phi_{core}\rangle_{r < R_{core}} - \langle\phi_{core}\rangle_{r > R_{core}}} \int_0^{+\infty} r^3 \frac{d\langle\phi_{core}(r)\rangle}{dr} dr \right]^{1/3} \quad (\text{S12})$$

where we used $\langle\phi_{core}\rangle_{r > R_{core}} = \max \langle\phi_{core}(r)\rangle$ and $\langle\phi_{core}\rangle_{r < R_{core}} - \langle\phi_{core}\rangle_{r > R_{core}} = -\langle\phi_{core}\rangle_{r > R_{core}}$.

The derivative and integral in Eq. S12 were evaluated numerically in the whole calculation box.

The total radius of the micelles R_H was obtained from the distribution of the neutral monomers $\langle\phi_N(r)\rangle$. Once again, we used the equimolar surface at R_H , defined by Equation S12.

Effect of the length of the guest molecule on the structure of C3Ms

Figure S1 shows the effect of varying the length of the guest molecule (z) on the radii of the micelle and the core, keeping constant the length of the copolymer blocks. The ratio of copolymer to guest molecules was varied in order to ensure a 1:1 stoichiometry between A and B blocks. The small scaling exponents obtained by fitting the experimental results (0.03 and 0.05, see Figure) are consistent with the nearly-zero scaling exponents experimentally found by Marras et al. for polyglutamic and DNA guest molecules combined with different

PEG-poly(lysine) diblock copolymers.⁹

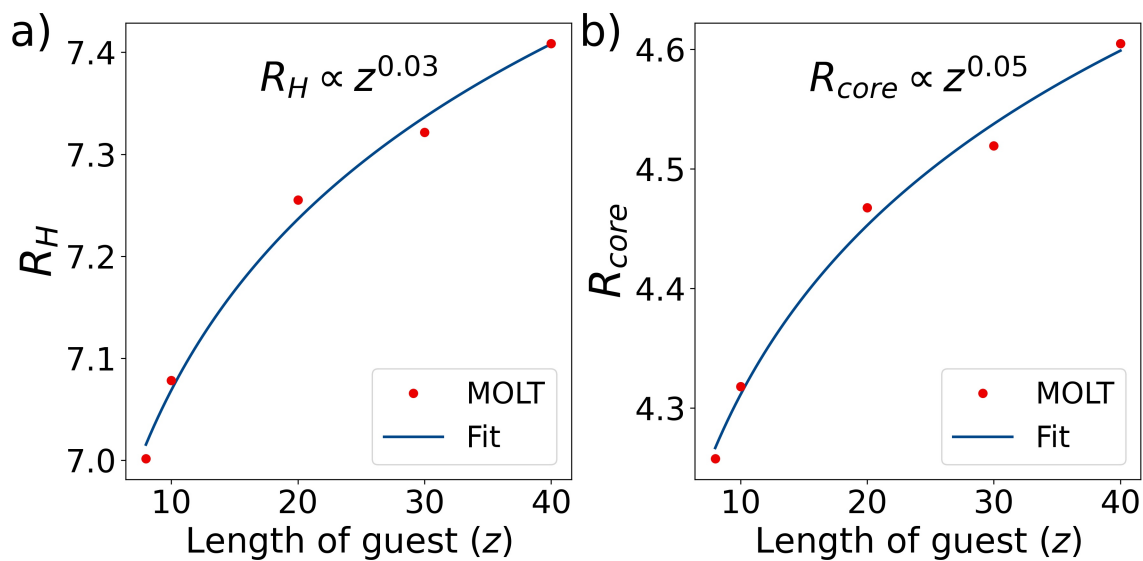


Figure S1: Effect of the length of the guest molecule B (for a fixed length of the charged block of the copolymer, A, and a 1:1 A:B fixed global stoichiometry on the radius of the micelle (left panel) and the core (right panel). Calculation parameters: length of the acidic copolymer block $x = 20$ and length of the neutral copolymer block $y = 20$.

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