Supporting information for "Droplet detachment force and its relation to Young-Dupre adhesion"

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Analytic solution for detachment force

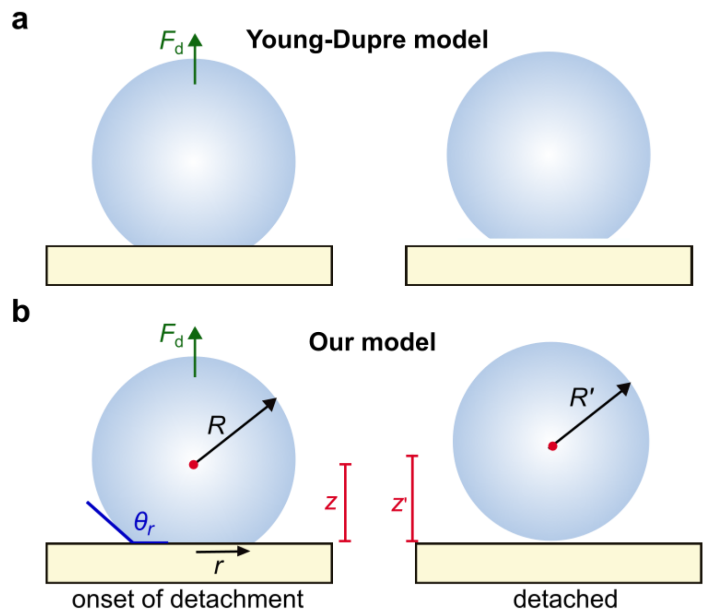

FIG. S1: Droplet detachment in **a)** Young-Dupre Model and **b)** our model.

Here, we will derive an expression for the non-dimensional detachment force $\tilde{F}_\text{d} = F_\text{d}/\gamma V^{1/3}$ as a power series of $\epsilon = (1 + \cos\theta_r)$. Our analytic solution applies for droplets with high contact angles, i.e., where $\epsilon \ll 1$.

At the onset of detachment, the droplet geometry can be modelled as a spherical cap of radius R, with the volume V and centroid position z given by

$$\begin{aligned} V &= \frac{\pi}{3}R^3(2+\cos\theta_r)(1-\cos\theta_r)^2 \\ &= \frac{\pi}{3}R^3(1+\epsilon)(2-\epsilon)^2 \end{aligned} \tag{S1}$$

and

$$\begin{aligned} z &= \frac{3R(1+\cos\theta_r)^2}{4(2+\cos\theta_r)} - R\cos\theta_r \\ &= \frac{3R\epsilon^2}{4(1+\epsilon)} - R(\epsilon-1) \\ &= R\left(1-\epsilon+\frac{3\epsilon^2}{4(1+\epsilon)}\right) \end{aligned} \tag{S2}$$

After detaching, the droplet is now a sphere with radius R' (which also equivalent to the new centroid position z') and by conservation of volume

$$\begin{aligned} \frac{4}{3}\pi R'^3 &= \frac{\pi}{3} R^3 (1+\epsilon)(2-\epsilon)^2 \\ R' &= R\frac{(1+\epsilon)^{1/3}(2-\epsilon)^{2/3}}{4^{1/3}} \end{aligned} \tag{S3}$$

The droplet's centroid position has been raised by an amount δz

$$\begin{aligned} \delta z &= z' - z \\ &= R\,\frac{(1+\epsilon)^{1/3}(2-\epsilon)^{2/3}}{4^{1/3}} - R\left(1-\epsilon+\frac{3\epsilon^2}{4(1+\epsilon)}\right) \\ &= R\left(\epsilon-\epsilon^2-\frac{2\epsilon^3}{3}+O(\epsilon^4)\right) \end{aligned} \tag{S4}$$

. There is also an increase in the surface area of the droplet by an amount δA, where

$$\begin{aligned} \delta A &= 4\pi R'^2 - 2\pi R^2(1-\cos\theta) \\ &= 4\pi R^2\,\frac{(1+\epsilon)^{2/3}(2-\epsilon)^{4/3}}{4^{2/3}} - 2\pi R^2(2-\epsilon) \\ &= 2\pi R^2\left(\epsilon-\epsilon^2+\frac{\epsilon^3}{3}+O(\epsilon^4)\right) \end{aligned} \tag{S5}$$

Total change in interfacial energy is given by $\Delta E_\gamma = \pi r^2(\gamma_s - \gamma_{ls}) + \delta A\,\gamma$, where γ is the droplet's surface tension, γ_s is the solid's surface energy, and γ_{ls} is the liquid-solid surface energy. In the Young-Dupre model, $\delta A = \pi r^2$. Here, we use the expression in Equation S5 for δA and the relations $r = R\sin\theta_r$ and $\gamma_s - \gamma_{ls} = \gamma\cos\theta_r$ to get

$$\begin{aligned} \Delta E_\gamma &= \pi R^2\gamma\sin^2\theta\cos\theta + 2\pi R^2\gamma\left(\epsilon-\epsilon^2+\frac{\epsilon^3}{3}+O(\epsilon^4)\right) \\ &= \pi R^2\gamma(1-\cos\theta)(1+\cos\theta)\cos\theta + 2\pi R^2\gamma\left(\epsilon-\epsilon^2+\frac{\epsilon^3}{3}+O(\epsilon^4)\right) \\ &= \pi R^2\gamma(-2\epsilon+3\epsilon^2-\epsilon^3) + 2\pi R^2\gamma\left(\epsilon-\epsilon^2+\frac{\epsilon^3}{3}+O(\epsilon^4)\right) \\ &= \pi R^2\gamma\left(\epsilon^2-\frac{\epsilon^3}{3}+O(\epsilon^4)\right) \end{aligned} \tag{S6}$$

The total change in interfaical energy ΔE_γ must be equivalent to the work done by the detachment force F_d over the distance δz, i.e.,

$$\begin{aligned}
F_d\,\delta z &= \Delta E_\gamma \\
F_d R &= \pi R^2 \gamma \left(\epsilon^2 - \frac{\epsilon^3}{3} + O(\epsilon^4)\right)\left(\epsilon - \epsilon^2 - \frac{2\epsilon^3}{3} + O(\epsilon^4)\right)^{-1} \\
F_d &= \pi R \gamma \left(\epsilon + \frac{2\epsilon^2}{3} + O(\epsilon^3)\right)
\end{aligned} \tag{S7}$$

We can recast Equation S1 to get $R = (3V/\pi)^{1/3}(1+\epsilon)^{-1/3}(2-\epsilon)^{-2/3}$ and substituting this to Equation S7 to get

$$\begin{aligned}
F_d &= \pi \left(\frac{3V}{\pi}\right)^{1/3} \gamma \left(\frac{\epsilon}{2^{2/3}} + O(\epsilon^2)\right) \\
\frac{F_d}{\gamma V^{1/3}} &= \left(\frac{\pi}{2}\right)^{2/3} 3^{1/3}\epsilon + O(\epsilon^2) \\
\tilde{F}_d &\approx \left(\frac{\pi}{2}\right)^{2/3} 3^{1/3}(1+\cos\theta) \\
&\approx 1.95(1+\cos\theta)
\end{aligned} \tag{S8}$$

Comparison between simulation and analytic results

Table S1: Comparison between simulation and analytic results. Body and surface $\tilde{F}_d$ values are obtained numerically by solving Young-Laplace equation, while analytic $\tilde{F}_d$ is calculated using Equation 5 in the main text. For $\theta > 140°$, $\tilde{F}_d$ values for all three simulation and analytic models are very close to one another (coloured cells).

θ	$1 + \cos\theta$	Body $\tilde{F}_d$	Surface $\tilde{F}_d$	Analytic $\tilde{F}_d$.
40°	1.77	5.31	1.56	3.44
50°	1.64	4.66	1.46	3.20
60°	1.50	4.01	1.37	2.92
70°	1.34	3.39	1.37	2.62
80°	1.17	2.80	1.35	2.28
90°	1	2.26	1.19	1.95
100°	0.83	2.80	1.35	2.28
110°	0.66	1.38	0.93	1.28
120°	0.5	1.02	0.76	0.97
130°	0.36	0.72	0.63	0.70
140°	0.23	0.46	0.39	0.46
150°	0.13	0.26	0.22	0.26
155°	0.093	0.18	0.18	0.18
160°	0.060	0.116	0.116	0.118
165°	0.034	0.066	0.067	0.066
170°	0.015	0.0295	0.0302	0.0296
175°	0.0038	0.0079	0.0077	0.0074