Supplementary Information

Emergence of a Yield Phenomenon in the Mechanics of Stochastic Networks with Inter-Fiber Cohesion

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1. Collapse of a stochastic network under the action of cohesive forces

The problem discussed in this section emerges from the methods used to generate networks with cohesion of type II. As discussed in the main text, a Voronoi network generated in a cubic domain of edge size L (State 1) is subjected to large cohesion forces which cause it to collapse. Specifically, the network undergoes an instability, after which it is stabilized again in a state of significantly smaller volume (about 24.5% of the original volume, L^3) by the formation of inter-fiber contacts. We provide here supplementary information related to this collapse process.

Cohesion forces produce in State 1 a hydrostatic stress state, characterized by a pressure $p^{adh}(\Psi)$. As Ψ is (artificially) increased to produce collapse, loss of stability occurs at a critical value of the pressure, p^{adh}_c, and a corresponding Ψ_c. Loss of stability may take place in the absence of cohesive forces, if the network is subjected to hydrostatic stress[1,2]. This leads to a response in compression characterized by an initial linear elastic regime, followed by an instability point and a plateau of small slope. The instability occurs at a critical applied pressure p^{appl}_c. Once inter-fiber contacts form at larger strains, the stress-strain curve exhibits strain stiffening. This phenomenology is also seen in cellular materials subjected to compression and p^{appl}_c is associated with the cell size and cell wall material properties[1]. It should be emphasized that in this situation, as well as in the case discussed in the main text, the behavior is entirely elastic as fibers are not allowed to deform plastically in this model.

Figure S1a shows the relationship between p^{appl}_c and p^{adh}_c evaluated with Voronoi networks of various w. w is the only relevant parameter in this context. The two critical pressures are approximately equal.

In both cases, the collapse is caused by the loss of stability of the structure and the associated mode involves many fibers and has complex topology. However, as shown in Fig. S1b, the data scales as

$$p^{appl}_c/E_f \sim f(w) = 10^{2w} = (\rho d^2)^2 \approx \left(d/l_{c0}\right)^4. \qquad \text{(S1)}$$

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In eq. (S1), the last equality is based on the result $\rho l_{c0}^2 \approx 1$, which was previously established for this type of networks[3].

Note that if one takes a mean field view and assumes the instability to be associated with the Euler buckling of individual fibers, $p_c/E_f \approx P_c/l_{c0}^2 E_f$, where P_c is the Euler buckling force and hence, using the expression for the buckling force P_c,

$$\frac{p_c}{E_f} \approx \frac{\pi^2 I_f}{l_{eff}^2} \frac{1}{l_{c0}^2} = \frac{\pi^3}{64\kappa^2}\left(\frac{d}{l_{c0}}\right)^4, \qquad \text{(S2)}$$

where the effective length is expressed in terms of the actual beam length through a shape factor, $l_{eff} = \kappa l_{c0}$. Requiring the constant of proportionality in eq. (S1) to be equal to $\pi^3/64\kappa^2$, the shape factor results $\kappa = 0.16$. This is smaller than the value of $\kappa = 0.5$ which corresponds to Euler buckling of a column with constrained rotations at both ends. This implies that collective instability takes place in the network at a larger stress than the prediction of this simplified mean field model but, interestingly, the scaling of the critical stress with d/l_{c0} reminiscent of the buckling of a single beam is retained.

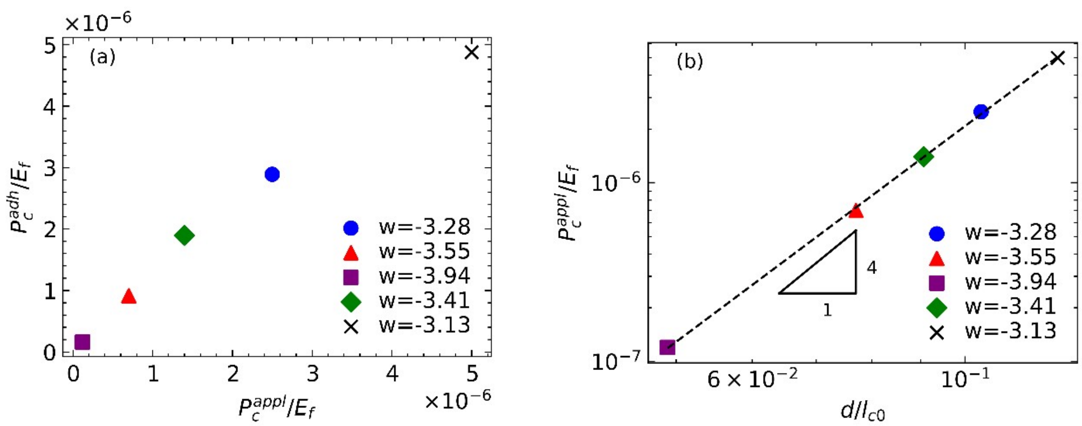

Figure S1. (a) Critical pressure produced by cohesion forces leading to the collapse of networks in State 1 and of different w, p_c^{adh}, shown versus the critical pressure applied at the boundary of the same networks without cohesion which causes collapse, p_c^{appl}. (b) Critical pressure p_c^{appl} scales as the fiber aspect ratio to power 4, as predicted by eq. (S2).

2. Inter-fiber contacts

Figure S2 shows the evolution of the number of contacts during deformation starting from State 2 of networks of types I and II. The number of contacts in type I networks is negligible (smaller than 5 in all cases). Type II networks form contacts even in the case without cohesion, $\Psi = 0$. These are reminiscent of the contact that stabilize the structure after collapse. Although these do not carry load in State 2 when $\Psi = 0$, they engage rapidly as deformation is applied. In cases with $\Psi > 0$, contacts exist in the unloaded State 2 and persist during the deformation. The most interesting aspect of the results shown in Fig. S2 is that the number of contacts is approximately constant throughout the deformation, despite that the population of contacts evolves, i.e. some contacts open and other form.

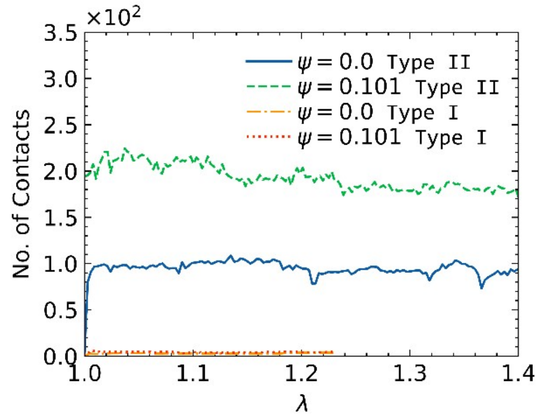

Figure S2. Number of contacts in the model for type I and type II networks with and without cohesion. The number of contacts in type I networks is smaller than 5 at all λ.

References

1 L. J. Gibson and M. F. Ashby, *Cellular Solids: Structure and Properties*, Cambridge University Press, Cambridge, 1997.

2 M. R. Islam and R. C. Picu, *J Appl Mech*, 2015, **85**, 85(8): 081011.

3 S. Deogekar, Z. Yan and R. C. Picu, *J Appl Mech*, 2019, **86**, 86(8): 081010.