## Supplementary Information for

The effects of asymmetry in active noises on the efficiency of single colloidal Stirling engines with active noises

## I. THE EFFECT OF CUTOFF ON THE MICROSCOPIC EFFICIENCY

We investigate the dependence of  $\langle \varepsilon \rangle$  on the cutoff value of the microscopic efficiency. When the cutoff value is between 3 and 1000, our simulation results for  $\langle \varepsilon \rangle$  are almost constant and robust. On the other hand, the values of  $\langle \varepsilon \rangle$  are fluctuating when cutoff is larger than 1000. The microscopic efficiency depends on a cutoff because of unconventionally large efficiencies. In this study, we use cutoff of 5 to avoid unconventionally large efficiencies.



Figure S1. The dependence of  $\langle \varepsilon \rangle$  on the cutoff values when  $\tau_a = 100$ ,  $T_a^c = 10000$ , and  $T_a^h = 40000$ .

## II. THE PERFORMANCE OF ENGINE WITH ACTIVE NOISES AS A FUNCTION OF $\tau_a$ PREDICTED FROM THE EFFECTIVE TEMPERATURE

Figures S2 (a), (b), and (c) show the ratios of  $W_s/W_q$ ,  $Q_{h,s}/Q_{h,q}$ , and  $W_s/Q_{h,s}$  as a function of  $\tau_a$ , respectively.  $W_q$  and  $Q_{h,q}$  are the output work and the absorbed heat of passive engines, respectively. As shown in Figures S2 (a) and (b), both  $W_s$  and  $Q_{h,s}$  increase with increasing  $\tau_a$  when  $T_a^h = 4T_a^c$  and  $T_a^h = 10T_a^c$ . But, the increase of  $W_s$  is much larger than that of  $Q_{h,s}$ . For example, when  $T_a^h = 4T_a^c$ , the value of  $W_s$  at  $\tau_a = 100$  increases about 84 times compared to  $W_q$ . On the other hand, the value of  $Q_{h,s}$  at  $\tau_a = 100$  increases about 26 times compared to  $Q_{h,s}$ . This difference makes the value of the macroscopic efficiency at  $\tau_a = 100$  increase by a factor of about three compared to  $-W_q/Q_{h,q}$  (Figure S2 (c)).

Also, Figures S2(a) and (b),  $W_s$  and  $Q_{h,s}$  converge to constant values as the value of  $\tau_a$  increases. Therefore, both macroscopic efficiency and microscopic efficiency also converge to constant values when the value of  $\tau_a$  is sufficiently large.



Figure S2. The ratio of (a)  $W_s/W_q$  and (b)  $Q_{h,s}/Q_{h,q}$  as a function of  $\tau_a$  when  $T_a^h = T_a^c$  (blue),  $T_a^h = 4T_a^c$  (black), and  $T_a^h = 10T_a^c$  (red). (c) The macroscopic efficiency predicted from  $W_s$  and  $Q_{h,s}$  as a function of  $\tau_a$  when  $T_a^h = T_a^c$  (blue),  $T_a^h = 4T_a^c$  (black), and  $T_a^h = 10T_a^c$  (red).