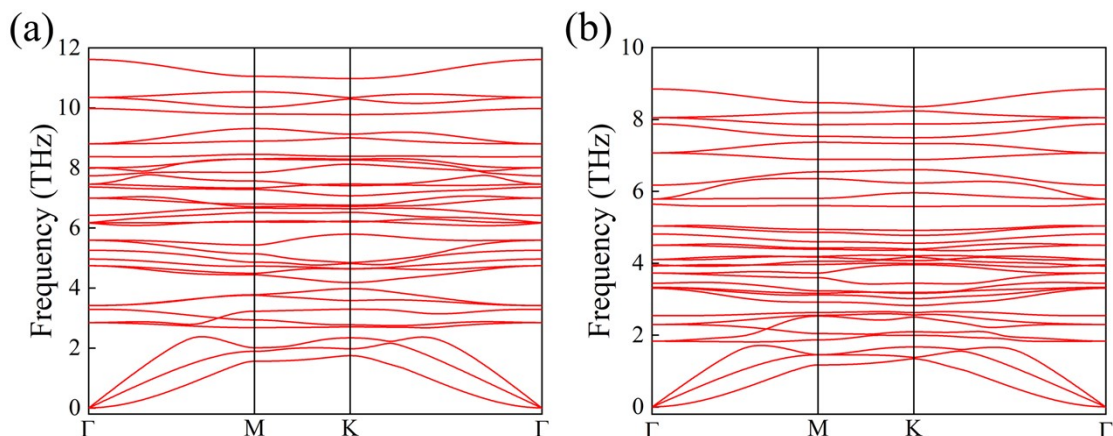


Supplementary materials for  
Enabling triferroics coupling in breathing kagome  
lattice  $\text{Nb}_3\text{X}_8$  ( $\text{X} = \text{Cl}, \text{Br}, \text{I}$ ) monolayers

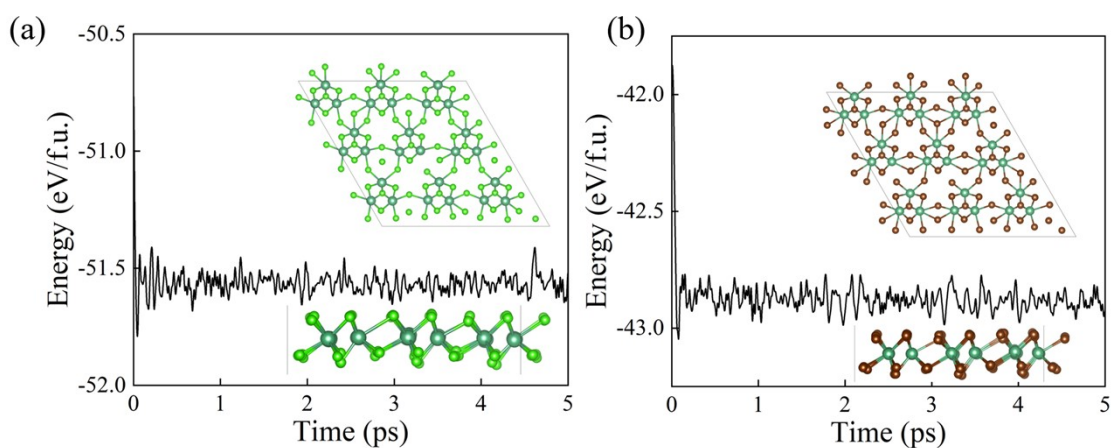
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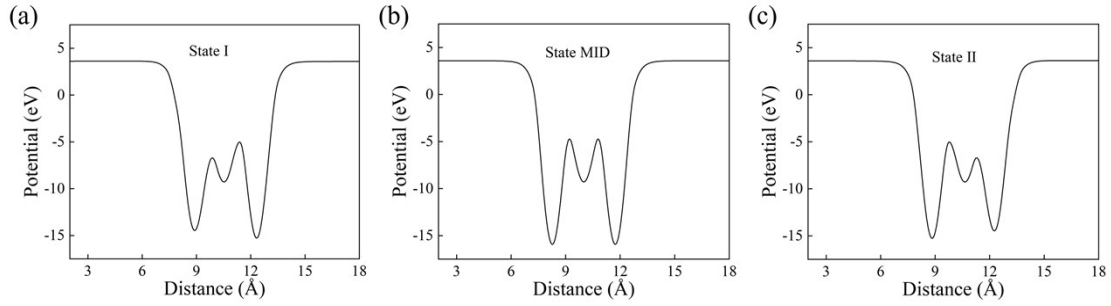
<sup>b</sup>Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China



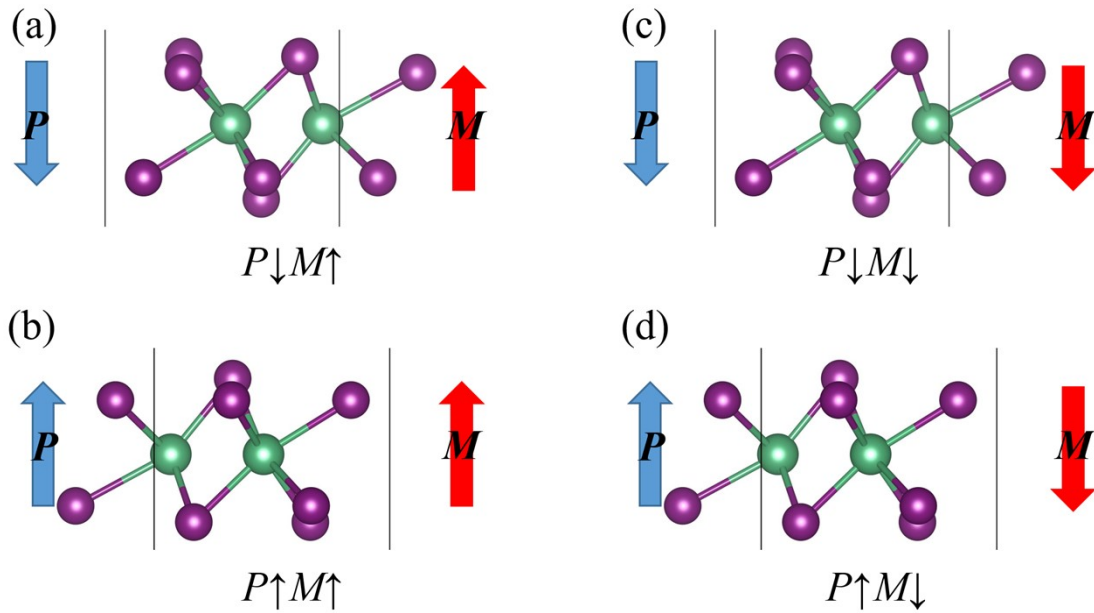
**Fig. S1** Phonon dispersion spectrums for the State I and State II of (a)  $\text{Nb}_3\text{Cl}_8$  and (b)  $\text{Nb}_3\text{Br}_8$  monolayers.



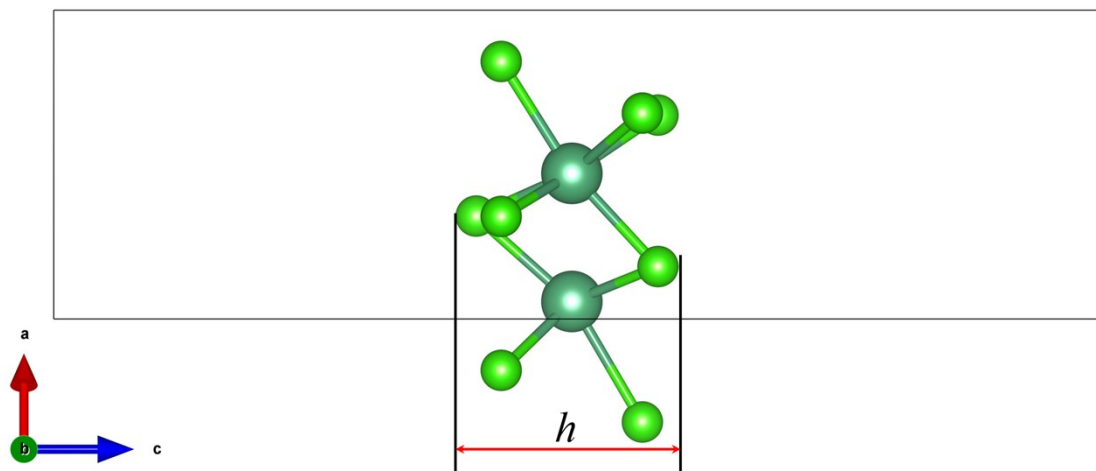
**Fig. S2** AIMD simulations of (a)  $\text{Nb}_3\text{Cl}_8$  and (b)  $\text{Nb}_3\text{Br}_8$  monolayers. The top and side views of final structure are in the picture.



**Fig. S3** Work functions along the  $z$  axis of ferroelectric and paraelectric phases: (a) State I; (b) State MID; (c) State II.

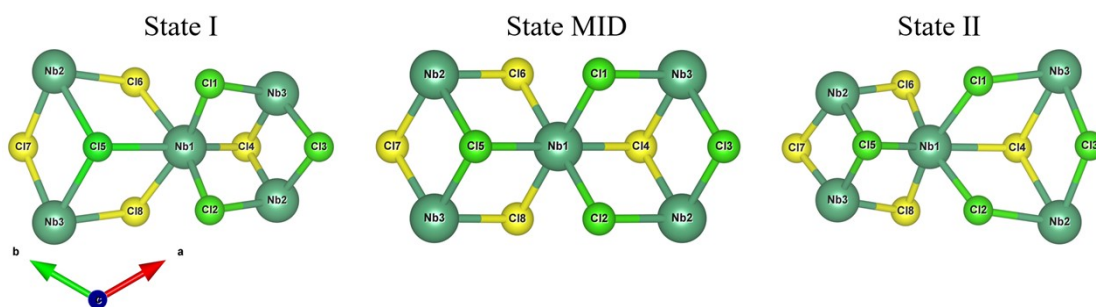


**Fig. S4** Four different FE/FM configurations of the  $\text{Nb}_3\text{I}_8$  monolayer: (a)  $P\downarrow M\uparrow$  configuration; (b)  $P\uparrow M\uparrow$  configuration; (c)  $P\downarrow M\downarrow$  configuration; (d)  $P\uparrow M\downarrow$  configuration. Blue and red arrows represent FE polarization and spin polarization directions, respectively.



**Fig. S5** Side view of the  $\text{Nb}_3\text{Cl}_8$  monolayer with periodic boundaries indicated by a black rectangle.  $h$  is defined as the thickness of the monolayer and the vacuum region of 20 Å along  $c$  direction is

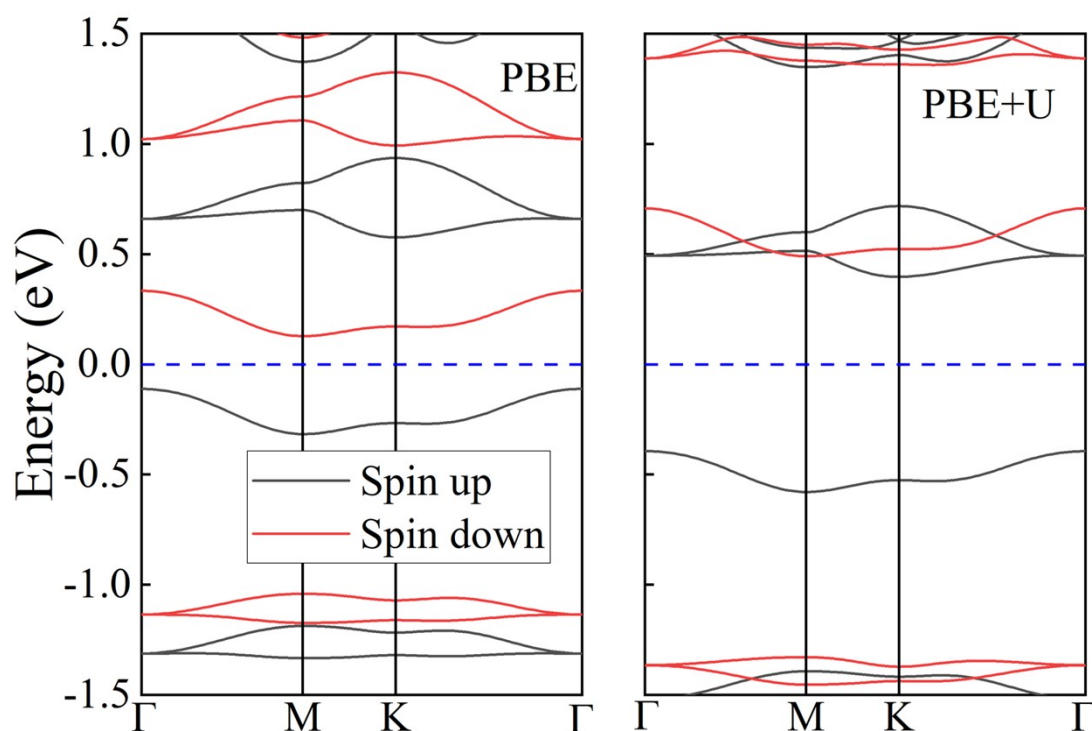
introduced in our calculations.



**Fig. S6** Top views of the  $\text{Nb}_3\text{Cl}_8$  monolayer for the State I, Stats MID and State II. The light green and yellow balls represent the Cl atoms at upper and bottom layers, respectively.

**Table S1.** Charge analysis of the  $\text{Nb}_3\text{Cl}_8$  monolayer by Hirshfeld method. The unit is  $e$ .

Atoms	State I	State MID	State II
$\text{Cl}_1, \text{Cl}_2, \text{Cl}_3$	-0.1089	-0.1367	-0.1587
$\text{Cl}_5$	-0.1077	-0.0843	-0.0266
$\text{Cl}_4$	-0.0226	-0.0843	-0.1077
$\text{Cl}_6, \text{Cl}_7, \text{Cl}_8$	-0.1587	-0.1367	-0.1089



**Fig. S7** Band structures of the  $\text{Nb}_3\text{Cl}_8$  monolayers by PBE or PBE+U calculations.