Measurement of fluid viscosity based on pressure-driven flow digital-printed microfluidics

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Supplementary Material

A. The relationship between the average flow velocity and the central linear flow velocity

Momentum equation for one-dimensional constant incompressible flow considering slit-type cross section，

$$\mu \frac{\partial^2 u}{\partial z^2} = \frac{\partial P}{\partial x} \tag{1}$$

- μ : Dynamic viscosity of the fluid.
- u : Velocity of the fluid in the x-direction.
- z : Coordinate perpendicular to the flow direction.
- $\frac{\partial P}{\partial x}$: Pressure gradient in the flow direction x.

integrating the equation,

$$u(z) = \frac{1}{2\mu}(-\frac{\partial P}{\partial x})z^2 + C_1 z + C_2 \tag{2}$$

Where C_1 and C_2 are integration constants. Bringing in the boundary conditions $u(\pm\frac{H}{2}) = 0$. H is height of the channel.

Determining the constant of integration, the flow velocity distribution is obtained as,

$$u(z) = \frac{1}{2\mu}(-\frac{\partial P}{\partial x})(\frac{H^2}{4} - z^2) \tag{3}$$

where $z = 0$, taking the maximum flow velocity at the centerline of the flow channel.

$$u_{\max} = \frac{H^2}{8\mu}(-\frac{\partial P}{\partial x}) \tag{4}$$

The average flow velocity of the flow channel is that,

$$u_{avg} = \frac{1}{H}\int_{-2/H}^{2/H} \frac{1}{2\mu}(-\frac{\partial P}{\partial x})(\frac{H^2}{4} - z^2)dz \tag{5}$$

integrating the equation,

$$u_{avg} = \frac{H^2}{12\mu}(-\frac{\partial P}{\partial x}) \tag{6}$$

The relationship between average flow velocity and centerline flow velocity is，

$$\mathrm{v(t)} = u_{\mathrm{avg}} = \frac{2}{3}u_{\mathrm{max}} \tag{7}$$

B. Darcy friction factor

From Eq. (6) in the main text, the pressure drop versus flow velocity for a slit-type flow channel cross-section is given by，

$$\Delta p = \frac{12\mu LQ}{wh^3} \tag{8}$$

- Δp : Pressure drop along the flow channel.
- μ : Dynamic viscosity of the fluid.
- w : Width and height of the flow channel.
- h : Height of the flow channel.
- Q : Flow rate.

Flow rate Q=vA=whv, v is the average flow velocity,

$$\Delta p = \frac{12\mu vL}{h^2} \tag{9}$$

combining the Darcy-Weisbach equation,

$$\Delta p = \rho h_f = \lambda \frac{L}{D_e}\frac{\rho v^2}{2g} \tag{10}$$

- ρ : Density of the fluid.
- h_f : Darcy friction factor.
- w : Width and height of the flow channel.

where D_e is the hydraulic radius, which is defined as 4 times the cross-sectional area divided by the wetted circumference and can be expressed as,

$$D_e = \frac{4A}{P} = \frac{2wh}{w+h} \tag{11}$$

when the rectangular section is a slit section, that is, shape, i.e. w is much larger than h, then $w + h \approx w$, can be simplified to $D_e = 2h$.

Compare (8) with (9), get

$$\lambda = \frac{48\mu}{\rho v H^2} \tag{12}$$

Combined with Reynolds number, the expression formula is obtained.

$$\mathrm{Re} = \frac{\rho v D_e}{\mu} \tag{13}$$

Darcy friction factor for rectangular cross-section with a large aspect ratio is

$$\lambda = \frac{96}{\mathrm{Re}} \tag{14}$$

C. Figure S 1

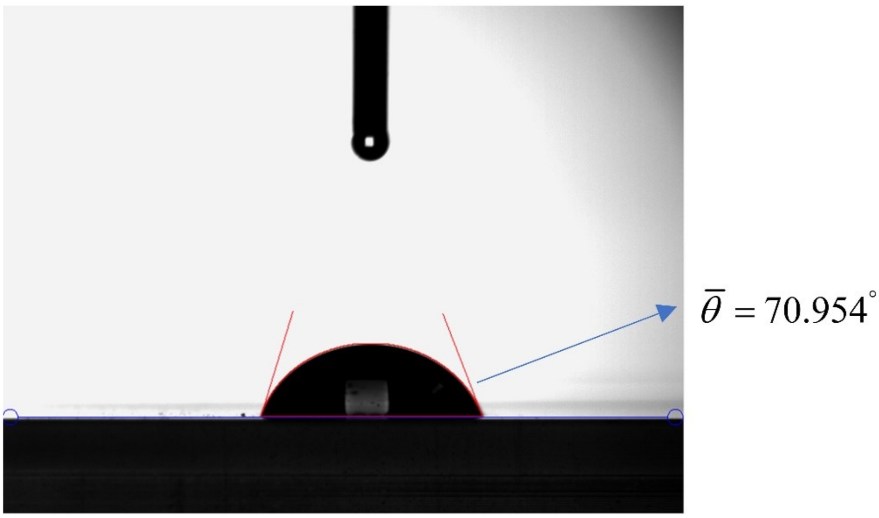

Figure S 1. Measurement results of contact angle

D. Figure S 2

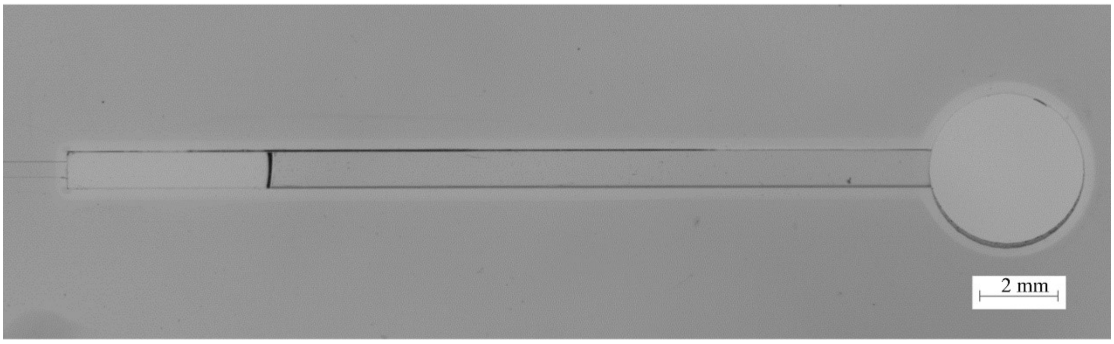

Figure S 2. liquid flow sequence

E. Figure S 3

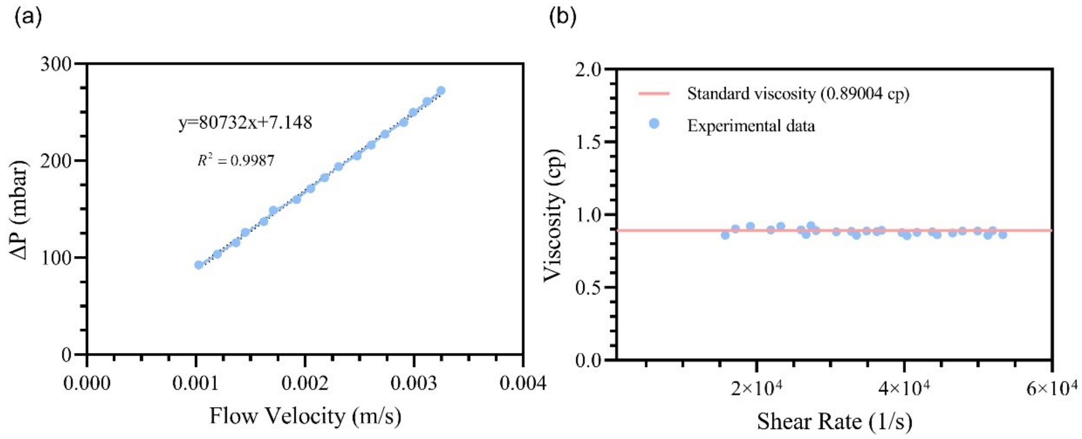

Figure S 3. The viscosity of water. (a) Pressure versus flow velocity relationship graph. The linear relationship indicates the Newtonian fluid characteristics of water. (b) Measured viscosity of water at 25° C, with the geometric correction factor allowing the measured value to match the standard value of 0.89004 cP.