

Supplementary Information (SI)

A reinvestigation of the boron cluster $B_{15}^{+0/-}$: A benchmark of density functionals and consideration of aromaticity models

**Yassin A. Jeilani,¹ Long Van Duong,^{2,4,*} Obaid Moraya Saeed Al Qahtani,¹
Minh Tho Nguyen^{3,4}**

¹ Department of Chemistry, University of Hail, Hail, Saudi Arabia

² Atomic Molecular and Optical Physics Research Group, Science and Technology Advanced Institute,
Van Lang University, Ho Chi Minh City, Vietnam

³ Laboratory of Chemical Computation and Modeling, Institute for Computational Science and Artificial
Intelligence, Van Lang University, Ho Chi Minh City, Vietnam

⁴ Faculty of Applied Technology, School of Technology, Van Lang University, Ho Chi Minh City, Vietnam

* Corresponding author: duongvanlong@vlu.edu.vn

Tables of supplementary information (SI)

Table S1. The T1 diagnostic of CCSD(T)/aug-cc-pVTZ single point of B₁₅^{+0/-}.

Isomer	T1	Isomer	T1	Isomer	T1	Isomer	T1
1a	0.036	2a	0.023	3a	0.018	4a	0.023
5a	0.020	6a	0.023	7a	0.032	8a	0.024
1n	0.045	2n	0.029	3n	0.031	4n	0.021
1c	0.020	2c	0.022	3c	0.021	4c	0.020

Table S2. Relative energies (kcal/mol) of ΔE_{3a-2a} , ΔE_{7a-2a} and ΔE_{4a-2a} and their errors $\Delta\Delta E_{3a-2a}$, $\Delta\Delta E_{7a-2a}$ and $\Delta\Delta E_{4a-2a}$ of B₁₅⁻. The MAE is in the last column.

	ΔE_{3a-2a}	ΔE_{7a-2a}	ΔE_{4a-2a}	$\Delta\Delta E_{3a-2a}$	$\Delta\Delta E_{7a-2a}$	$\Delta\Delta E_{4a-2a}$	MAE
CCSD(T) ^c	2.4	6.3	5.5	-	-	-	-
B3LYP ^a	-1.19	0.83	12.95	-3.59	-5.47	7.45	5.50
B3LYP ^b	-1.71	0.30	12.10	-4.11	-6.00	6.60	5.57
M06 ^a	5.78	5.56	11.57	3.38	-0.74	6.07	3.40
M06 ^b	5.34	5.56	10.47	2.94	-0.74	4.97	2.88
VSXC ^a	1.16	1.65	10.56	-1.24	-4.65	5.06	3.65
VSXC ^b	0.51	1.21	9.72	-1.89	-5.09	4.22	3.73
B1B95 ^a	4.04	6.10	3.87	1.64	-0.20	-1.63	1.16
B1B95 ^b	3.69	5.71	3.19	1.29	-0.59	-2.31	1.40
BPBE ^a	0.55	1.41	9.30	-1.85	-4.89	3.80	3.51
BPBE ^b	0.15	1.02	8.46	-2.25	-5.28	2.96	3.50
TPSSh ^a	0.25	1.44	5.16	-2.15	-4.86	-0.34	2.45
TPSSh ^b	-0.29	1.02	4.36	-2.69	-5.28	-1.14	3.04
PBE ^a	1.50	2.26	9.04	-0.90	-4.04	3.54	2.83
PBE ^b	1.02	1.83	8.24	-1.38	-4.47	2.74	2.86
PBE0 ^a	4.02	2.61	7.83	1.62	-3.69	2.33	2.55
PBE0 ^b	3.55	2.22	7.00	1.15	-4.08	1.50	2.24
HSE06 ^a	3.40	2.18	8.41	1.00	-4.12	2.91	2.68
HSE06 ^b	2.96	1.81	7.55	0.56	-4.49	2.05	2.37
B2PLYPD3 ^a	2.44	4.59	2.44	0.04	-1.71	-3.06	1.60
B2PLYPD3 ^b	3.07	4.52	2.52	0.67	-1.78	-2.98	1.81

^{a)} The DFT functional in conjunction with the 6-311+G(d) basis set, ^{b)} The DFT functional in conjunction with the def2-TZVPP basis set, ^{c)} The singlet-point CCSD(T)/aug-cc-pVTZ references values.

Table S3. Relative energies (kcal/mol) of ΔE_{2n-1n} , ΔE_{3n-1n} and ΔE_{4n-1n} and their errors $\Delta\Delta E_{2n-1n}$, $\Delta\Delta E_{3n-1n}$ and $\Delta\Delta E_{4n-1n}$ of B₁₅. The MAE is in the last column.

	ΔE_{2n-1n}	ΔE_{3n-1n}	ΔE_{4n-1n}	$\Delta\Delta E_{2n-1n}$	$\Delta\Delta E_{3n-1n}$	$\Delta\Delta E_{4n-1n}$	MAE
CCSD(T) ^c	1.9	3.9	16.0	-	-	-	-
B3LYP ^a	2.40	0.65	42.02	0.50	-3.25	26.02	15.14
B3LYP ^b	2.22	0.54	41.87	0.32	-3.36	25.87	15.06
M06 ^a	3.04	3.60	3.26	1.14	-0.30	-12.74	7.39
M06 ^b	2.09	3.50	2.11	0.19	-0.40	-13.89	8.03
VSXC ^a	2.77	1.83	26.14	0.87	-2.07	10.14	6.00
VSXC ^b	2.38	1.39	26.07	0.48	-2.51	10.07	6.00
B1B95 ^a	1.82	3.51	-6.93	-0.08	-0.39	-22.93	13.24
B1B95 ^b	-0.92	3.27	-7.05	-2.82	-0.63	-23.05	13.41
BPBE ^a	2.73	2.85	22.30	0.83	-1.05	6.30	3.72
BPBE ^b	2.42	2.63	21.86	0.52	-1.27	5.86	3.47
TPSSh ^a	1.52	1.68	10.05	-0.38	-2.22	-5.95	3.68
TPSSh ^b	1.16	1.37	9.78	-0.74	-2.53	-6.22	3.90
PBE ^a	2.94	3.49	18.45	1.04	-0.41	2.45	1.56
PBE ^b	2.61	3.22	18.18	0.71	-0.68	2.18	1.38
PBE0 ^a	2.15	3.74	10.43	0.25	-0.16	-5.57	3.22
PBE0 ^b	1.87	3.56	10.24	-0.03	-0.34	-5.76	3.33
HSE06 ^a	2.32	3.43	12.50	0.42	-0.47	-3.50	2.05
HSE06 ^b	2.01	3.24	12.22	0.11	-0.66	-3.78	2.21
B2PLYPD3 ^a	2.06	1.42	37.10	0.16	-2.48	21.10	12.26
B2PLYPD3 ^b	1.91	1.29	36.74	0.01	-2.61	20.74	12.07

^{a)} The DFT functional in conjunction with the 6-311+G(d) basis set, ^{b)} The DFT functional in conjunction with the def2-TZVPP basis set, ^{c)} The singlet-point CCSD(T)/aug-cc-pVTZ references values.

Table S4. Relative energies (kcal/mol) of ΔE_{2c-1c} , ΔE_{3c-1c} and ΔE_{4c-1c} and their errors $\Delta\Delta E_{2c-1c}$, $\Delta\Delta E_{3c-1c}$ and $\Delta\Delta E_{4c-1c}$ of B_{15}^+ . The MAE is in the last column.

	ΔE_{2c-1c}	ΔE_{3c-1c}	ΔE_{4c-1c}	$\Delta\Delta E_{2c-1c}$	$\Delta\Delta E_{3c-1c}$	$\Delta\Delta E_{4c-1c}$	MAE
CCSD(T) ^c	1.9	3.9	16.0	-	-	-	-
B3LYP ^a	2.40	0.65	42.02	0.50	-3.25	26.02	15.14
B3LYP ^b	2.22	0.54	41.87	0.32	-3.36	25.87	15.06
M06 ^a	3.04	3.60	3.26	1.14	-0.30	-12.74	7.39
M06 ^b	2.09	3.50	2.11	0.19	-0.40	-13.89	8.03
VSXC ^a	2.77	1.83	26.14	0.87	-2.07	10.14	6.00
VSXC ^b	2.38	1.39	26.07	0.48	-2.51	10.07	6.00
B1B95 ^a	1.82	3.51	-6.93	-0.08	-0.39	-22.93	13.24
B1B95 ^b	-0.92	3.27	-7.05	-2.82	-0.63	-23.05	13.41
BPBE ^a	2.73	2.85	22.30	0.83	-1.05	6.30	3.72
BPBE ^b	2.42	2.63	21.86	0.52	-1.27	5.86	3.47
TPSSh ^a	1.52	1.68	10.05	-0.38	-2.22	-5.95	3.68
TPSSh ^b	1.16	1.37	9.78	-0.74	-2.53	-6.22	3.90
PBE ^a	2.94	3.49	18.45	1.04	-0.41	2.45	1.56
PBE ^b	2.61	3.22	18.18	0.71	-0.68	2.18	1.38
PBE0 ^a	2.15	3.74	10.43	0.25	-0.16	-5.57	3.22
PBE0 ^b	1.87	3.56	10.24	-0.03	-0.34	-5.76	3.33
HSE06 ^a	2.32	3.43	12.50	0.42	-0.47	-3.50	2.05
HSE06 ^b	2.01	3.24	12.22	0.11	-0.66	-3.78	2.21
B2PLYPD3 ^a	2.06	1.42	37.10	0.16	-2.48	21.10	12.26
B2PLYPD3 ^b	1.91	1.29	36.74	0.01	-2.61	20.74	12.07

^{a)} The DFT functional in conjunction with the 6-311+G(d) basis set, ^{b)} The DFT functional in conjunction with the def2-TZVPP basis set, ^{c)} The singlet-point CCSD(T)/aug-cc-pVTZ references values.

Figures of supplementary information (SI)

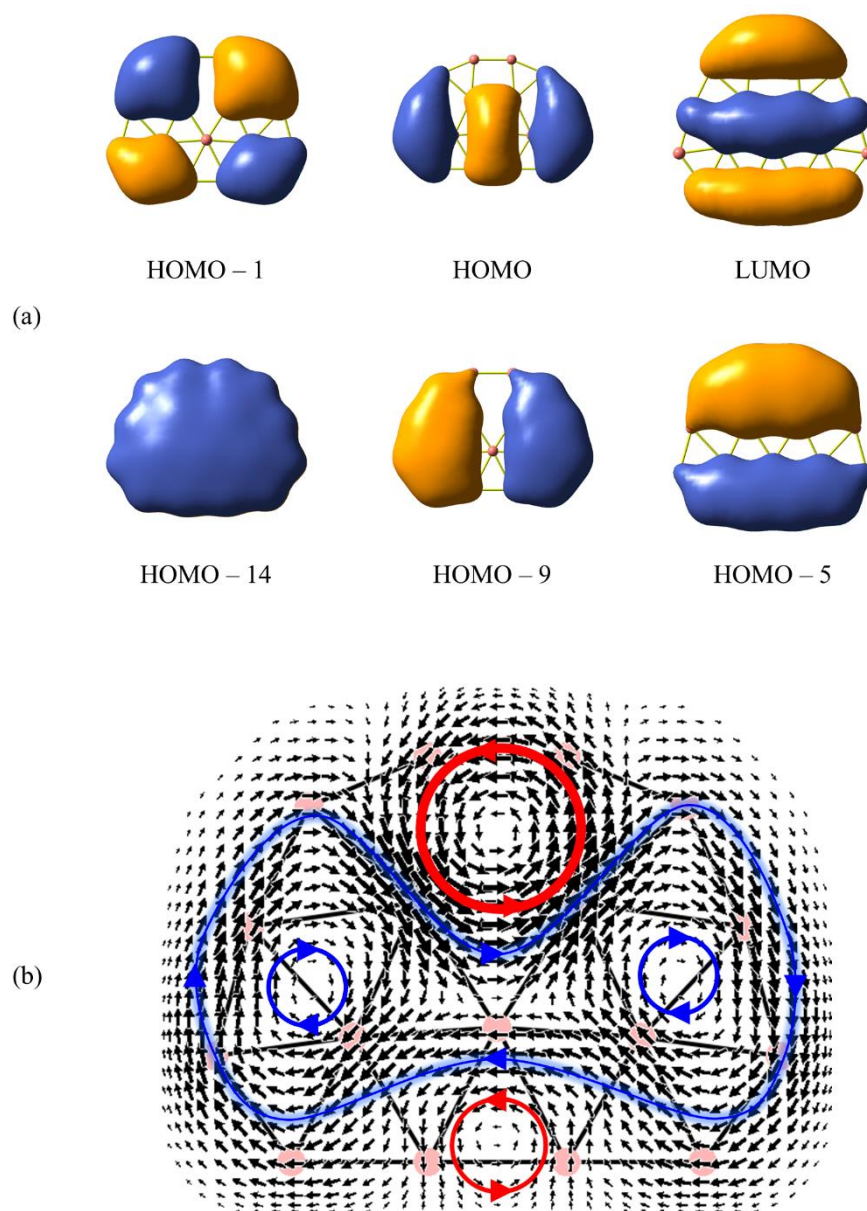


Figure S1. (a) The π -MOs and (b) the π ring current map of B_{17}^- . Local diatropic and local paramagnetic ring currents are highlighted by blue and red rings, respectively.

SI: The python code

The python code to compute the dimensions $L_a \times L_b$ of a rectangle box and the root mean square error (RMSE) (namely rectanglebox.py):

```
import matplotlib.pyplot as plt
import numpy as np
import math
from numpy import random
import scipy.special as sp
import scipy.optimize as opt

def find_La_Lb(C,abE):
    E_RM = (abE[:,0]**2)*C[0] + (abE[:,1]**2)*C[1]
    E_dft = E_RM*C[2] + C[3]
    return np.sum((E_dft - abE[:,2])**2)

plt.rcParams["figure.figsize"] = (5,5)
Edft = np.loadtxt("aC2v.txt", usecols=(0,1,2))
n_occ = 5
n_unocc = 3
xmin = math.floor(100*(np.min(Edft[:, 2]) - 0.02))/100
xmax = math.ceil(100*(np.max(Edft[:, 2]) + 0.02))/100
x = np.linspace(xmin, xmax, 11)
y = x
plt.plot(x, y,'k--')
plt.axis('equal')

c_guess = np.ones(4)
c_guess[3] = -1

Eocc = Edft[:n_occ, :]
Eunocc = Edft[-n_unocc:, :]
Cminimo = opt.minimize(find_La_Lb,c_guess,args=(Eocc))
c_out = Cminimo.x
La = 0.529177249*np.pi/np.sqrt(2*c_out[0]*c_out[2])
Lb = 0.529177249*np.pi/np.sqrt(2*c_out[1]*c_out[2])
E_1 = (Eocc[:,0]**2)*c_out[0] + (Eocc[:,1]**2)*c_out[1]
E_r_occ = E_1*c_out[2] + c_out[3]
RMSE = np.sqrt(np.sum((E_r_occ - Eocc[:,2])**2)/n_occ)
E_2 = (Eunocc[:,0]**2)*c_out[0] + (Eunocc[:,1]**2)*c_out[1]
E_r_unocc = E_2*c_out[2] + c_out[3]
plt.plot(E_r_occ, Eocc[:,2], 'ro',markersize=12,fillstyle='none')
plt.plot(E_r_unocc, Eunocc[:,2], 'bs',markersize=12,fillstyle='none')
print(f"Rectangle box:")
print(f"    La = {La:.4f} Angstrom")
print(f"    Lb = {Lb:.4f} Angstrom")
print(f"    La/Lb = {La/Lb:.4f} ")
print(f"    RMSE = {RMSE:.4f} ")
print(f"    ")
```

```
plt.savefig('rectangle.png', dpi=300)
```

will read the MO information of **1a-C_{2v}** in the file “aC2v.txt”:

1	1	-0.212942157
2	1	-0.147224498
1	2	-0.127987704
3	1	-0.076135935
2	2	-0.034904343
1	3	-0.014415061
4	1	0.045922111
3	2	0.046338683

content with three columns representing the quantum numbers *a*, *b*, and MO energy (Hartree), respectively.

Additional information on isomers

Cartesian coordinates (in angstrom), energies (E, Hartree), and zero-point energies (ZPE, Hartree) of lower-lying isomers optimized at the PBE/6-311+G(d) level.

1a

E = -372.115403224, ZPE = 0.05783

5	-0.029089000	1.145113000	2.018757000
5	-0.101002000	-0.124567000	0.844625000
5	-0.101002000	-0.124567000	-0.844625000
5	0.052006000	-0.356989000	2.481697000
5	-0.302907000	-1.639556000	0.000000000
5	0.169519000	1.501723000	0.000000000
5	-0.046903000	2.587783000	1.387771000
5	-0.029089000	1.145113000	-2.018757000
5	0.111916000	3.223220000	0.000000000
5	-0.046903000	2.587783000	-1.387771000
5	0.012482000	-1.765314000	1.748570000
5	0.052006000	-0.356989000	-2.481697000
5	0.123241000	-3.028720000	-0.777840000
5	0.123241000	-3.028720000	0.777840000
5	0.012482000	-1.765314000	-1.748570000

1a-C_{2v}

E = -372.104077266; Number of imaginary frequencies = 2

5	0.000000000	1.987156000	1.157686000
5	0.000000000	0.831981000	-0.115526000
5	0.000000000	-0.831981000	-0.115526000
5	0.000000000	2.461528000	-0.367077000
5	0.000000000	0.000000000	-1.612398000
5	0.000000000	0.000000000	1.491774000
5	0.000000000	1.398267000	2.600467000
5	0.000000000	-1.987156000	1.157686000
5	0.000000000	0.000000000	3.234548000
5	0.000000000	-1.398267000	2.600467000
5	0.000000000	1.777568000	-1.790919000
5	0.000000000	-2.461528000	-0.367077000
5	0.000000000	-0.779837000	-3.041593000
5	-0.000000000	0.779837000	-3.041593000
5	0.000000000	-1.777568000	-1.790919000

2a

E = -372.1150576, ZPE = 0.060007

5	-0.905151000	1.484710000	0.085927000
5	-2.521820000	1.337327000	0.283473000
5	0.420672000	2.374192000	-0.363829000
5	-3.414754000	0.092577000	-0.054838000
5	-0.210561000	-0.369298000	0.216782000
5	-2.788013000	-1.348954000	-0.003908000
5	1.940809000	2.059005000	-0.211776000
5	-1.212331000	-1.707871000	-0.048346000
5	-1.768234000	-0.048161000	-0.320804000
5	2.706952000	0.679755000	0.053403000
5	0.831446000	0.982935000	0.398174000
5	3.044811000	-0.894104000	0.040617000
5	2.037124000	-2.083896000	-0.097765000
5	1.434431000	-0.593688000	0.164052000
5	0.404620000	-1.964529000	-0.141162000

3a

E = -372.1112236; ZPE = 0.057794

5	0.000000000	1.496216000	0.809929000
5	0.000000000	0.775607000	3.503188000
5	0.000000000	0.000000000	1.865603000
5	0.000000000	1.496216000	-0.809929000
5	0.000000000	0.000000000	0.000000000
5	0.000000000	0.000000000	-1.865603000
5	0.000000000	-1.496216000	-0.809929000
5	0.000000000	0.775607000	-3.503188000
5	0.000000000	-1.747941000	-2.323776000
5	0.000000000	-0.775607000	-3.503188000
5	0.000000000	1.747941000	-2.323776000
5	0.000000000	1.747941000	2.323776000
5	0.000000000	-1.747941000	2.323776000
5	0.000000000	-1.496216000	0.809929000
5	0.000000000	-0.775607000	3.503188000

4a

E = -372.1003049; ZPE = 0.058385

5	0.846014000	-1.878289000	0.246587000
5	-0.765450000	-1.919832000	-0.165139000
5	2.404071000	-1.579971000	-0.197393000
5	-2.877405000	-0.117223000	-0.679180000
5	-0.006934000	-0.552544000	0.978314000
5	-2.194824000	1.147776000	0.070816000
5	-0.977225000	0.848647000	1.160449000
5	-0.683477000	1.916180000	-0.215047000
5	-1.522815000	-0.429151000	0.208715000
5	2.916554000	-0.154518000	-0.511854000
5	1.572260000	-0.317377000	0.413381000
5	2.055070000	1.254064000	-0.476570000
5	-2.200836000	-1.555384000	-0.758700000
5	0.836960000	2.237915000	-0.630622000
5	0.598037000	1.099708000	0.556244000

7a

E = -372.1094639; ZPE = 0.057329

5	1.437965000	1.818120000	0.000000000
5	0.000000000	0.856403000	0.000000000
5	-0.178027000	-0.854047000	0.000000000
5	0.010183000	2.525251000	0.000000000
5	-1.589264000	0.191612000	0.000000000
5	1.535687000	-0.031232000	0.000000000
5	2.847994000	1.072090000	0.000000000
5	0.800083000	-2.311850000	0.000000000
5	3.164948000	-0.443276000	0.000000000
5	2.263997000	-1.691376000	0.000000000
5	-1.483061000	1.959544000	0.000000000
5	-0.739165000	-2.435986000	0.000000000
5	-3.136340000	-0.347316000	0.000000000
5	-2.894275000	1.195502000	0.000000000
5	-2.040723000	-1.503439000	0.000000000

1n

E = -371.9983039; ZPE = 0.057496

5	0.087491000	-0.111842000	0.834027000
5	0.087491000	-0.111842000	-0.834027000
5	0.159348000	-1.613954000	0.000000000
5	-0.045039000	-0.360303000	2.473948000
5	-0.063580000	3.230969000	0.000000000
5	-0.019394000	-1.781906000	-1.768842000
5	0.016741000	2.604263000	1.392239000
5	-0.061622000	-3.042801000	-0.772206000
5	-0.045039000	-0.360303000	-2.473948000
5	0.006086000	1.141441000	-2.002951000
5	0.006086000	1.141441000	2.002951000
5	-0.019394000	-1.781906000	1.768842000
5	-0.061622000	-3.042801000	0.772206000
5	-0.064296000	1.485281000	0.000000000
5	0.016741000	2.604263000	-1.392239000

2n

E = -371.9944239; ZPE = 0.057778

5	1.708828000	-0.146988000	2.371212000
5	1.708828000	-0.146988000	-2.371212000
5	1.957679000	-0.079749000	0.815514000
5	1.957679000	-0.079749000	-0.815514000
5	0.140722000	0.073371000	1.524789000
5	-1.051260000	-0.071887000	2.812425000
5	-1.851812000	-0.082413000	-1.457325000
5	-1.851812000	-0.082413000	1.457325000
5	0.421464000	-0.130394000	3.228233000
5	-2.434115000	-0.173922000	0.000000000
5	0.421464000	-0.130394000	-3.228233000
5	0.140722000	0.073371000	-1.524789000
5	-0.906516000	0.438462000	0.000000000
5	0.689387000	0.611581000	0.000000000
5	-1.051260000	-0.071887000	-2.812425000

3n

E = -371.9932837; ZPE = 0.057609

5	0.000000000	1.499362000	0.800986000
5	0.000000000	0.769733000	3.512854000
5	0.000000000	0.000000000	1.852530000
5	0.000000000	1.499362000	-0.800986000
5	0.000000000	0.000000000	0.000000000
5	0.000000000	0.000000000	-1.852530000
5	0.000000000	-1.499362000	-0.800986000
5	0.000000000	0.769733000	-3.512854000
5	0.000000000	-1.746338000	-2.326075000
5	0.000000000	-0.769733000	-3.512854000
5	0.000000000	1.746338000	-2.326075000
5	0.000000000	1.746338000	2.326075000
5	0.000000000	-1.746338000	2.326075000
5	0.000000000	-1.499362000	0.800986000
5	0.000000000	-0.769733000	3.512854000

4n

E = -371.9697324; ZPE = 0.057893

5	0.000000000	2.740992000	-0.760910000
5	0.000000000	0.000000000	0.982202000
5	-1.364664000	2.114714000	-0.345955000
5	-1.540502000	-0.833879000	0.525114000
5	0.000000000	0.874746000	-0.861337000
5	1.364664000	-2.114714000	-0.345955000
5	0.000000000	-2.740992000	-0.760910000
5	0.000000000	1.681438000	0.772830000
5	1.540502000	-0.833879000	0.525114000
5	-1.540502000	0.833879000	0.525114000
5	1.364664000	2.114714000	-0.345955000
5	1.540502000	0.833879000	0.525114000
5	0.000000000	-0.874746000	-0.861337000
5	0.000000000	-1.681438000	0.772830000
5	-1.364664000	-2.114714000	-0.345955000

1c

E = -371.7264283; ZPE = 0.057656

5	0.00000000	2.01192000	1.13340200
5	0.00000000	0.82840800	-0.09959000
5	0.00000000	-0.82840800	-0.09959000
5	0.00000000	2.48309900	-0.36252000
5	0.00000000	0.00000000	-1.60163400
5	0.00000000	0.00000000	1.48067700
5	0.00000000	1.39233600	2.62069800
5	0.00000000	-2.01192000	1.13340200
5	0.00000000	0.00000000	3.24588100
5	0.00000000	-1.39233600	2.62069800
5	0.00000000	1.78520400	-1.79682900
5	0.00000000	-2.48309900	-0.36252000
5	0.00000000	-0.76862100	-3.05762300
5	0.00000000	0.76862100	-3.05762300
5	0.00000000	-1.78520400	-1.79682900

2c

E = -371.722467; ZPE = 0.057787

5	-1.70154900	-0.17914900	2.37787500
5	-0.13170600	0.14585600	1.52577300
5	0.90031700	0.43647900	0.00000000
5	2.45453100	-0.13326700	0.00000000
5	-0.69747100	0.62399400	0.00000000
5	-1.96901800	-0.09019200	-0.80356200
5	-0.42140100	-0.11159000	3.24343300
5	1.85212600	-0.10779400	1.44536700
5	-0.13170600	0.14585600	-1.52577300
5	1.04286000	-0.12073300	2.82349800
5	-1.96901800	-0.09019200	0.80356200
5	1.85212600	-0.10779400	-1.44536700
5	-0.42140100	-0.11159000	-3.24343300
5	-1.70154900	-0.17914900	-2.37787500
5	1.04286000	-0.12073300	-2.82349800

3c

E = -371.7149238; ZPE = 0.057062

5	0.00000000	1.50488300	0.79465900
5	0.00000000	0.76664200	3.53425700
5	0.00000000	0.00000000	1.84247600
5	0.00000000	1.50488300	-0.79465900
5	0.00000000	0.00000000	0.00000000
5	0.00000000	0.00000000	-1.84247600
5	0.00000000	-1.50488300	-0.79465900
5	0.00000000	0.76664200	-3.53425700
5	0.00000000	-1.74741800	-2.33572100
5	0.00000000	-0.76664200	-3.53425700
5	0.00000000	1.74741800	-2.33572100
5	0.00000000	1.74741800	2.33572100
5	0.00000000	-1.74741800	2.33572100
5	0.00000000	-1.50488300	0.79465900
5	0.00000000	-0.76664200	3.53425700

4c

E = -371.7171278; ZPE = 0.058825

5	0.00000000	2.03584100	0.00000000
5	1.60194900	1.82288200	0.00000000
5	-1.60194900	1.82288200	0.00000000
5	0.00000000	-0.99386300	0.90784700
5	0.86071100	0.49693200	-0.90784700
5	2.37963600	0.47588800	0.00000000
5	1.76309000	-1.01792000	0.00000000
5	-0.86071100	0.49693200	0.90784700
5	0.86071100	0.49693200	0.90784700
5	-2.37963600	0.47588800	0.00000000
5	-0.86071100	0.49693200	-0.90784700
5	-1.76309000	-1.01792000	0.00000000
5	-0.77768700	-2.29876900	0.00000000
5	0.00000000	-0.99386300	-0.90784700
5	0.77768700	-2.29876900	0.00000000