## Supplementary material

1- Comparisons with previously numerical computations based on the dipole discrete approximation (DDA):



**Fig. 1S.** The calculated extinction spectra of silica core – gold shell rod-shaped nanoshells immersed in water, as predicted by our approach; (a) the longitudinal modes, and (b) the transverse modes. The extinction spectra are plotted for four structural design cases assumed in [54] for which  $2a_x = 2a_y = 10 nm_{and} 2a_z = 80 nm_{(i.e., the symbols} 2a_x, 2a_z, and d in Fig. 1(b) correspond to S_o, L_o, and <math>(L_o - L_i)/2$  in [54]). The values of  $r_1$  and d are displayed in this figure for all cases. For longitudinal modes of the first and second case study when  $(r_1,d) = (24 nm, 16 nm)$  and (20 nm, 20 nm), respectively, the results of our approach match very well with the DDA results published in [54]. For the third and fourth case study when  $(r_1,d) = (16 nm, 24 nm)$  and

(14 nm, 26 nm), respectively,  $\lambda_{res}$  obtained by our approach begins to diverge from that obtained by the DDA method where the shell thickness d (particularly of the fourth case) greatly exceeds the limitation range proposed in our study ( $d_{max} \leq 12.5$ ). For transverse modes, a very good match between our results and those of the DDA method has been realized for all cases.



## 2- Comparisons with previously experimental results:

**Fig. 2S.** The calculated extinction spectra of Au nanorods coated with silica and immersed in ethanol, as anticipated by our approach. The geometrical parameters of nanoshells are assumed as given in [56] where the Au nanorods have longitudinal and transverse diameters of 52.5 nm and 15 nm, respectively (i.e., using our symbols,  $2a_z = 52.5 nm$  and  $2a_x = 2a_y = 15 nm$ ). After coating with silica, the shell thicknesses of silica were 10 nm, 12 nm, and 14 nm. There is good agreement between the peak position of the absorption bands of the silica-coated Au nanorods, as estimated by our approach, and the observed ones reported in Ref. [56].