Supporting Information for:

Modeling Separation of Lanthanides via Heterogeneous Ligand Binding

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Contents

S1. EFFECT OF VAN DER WAAL'S CORRECTIONS (DFT-D3)

To address the possible effect of dispersion forces, not included in the main text, spot checks using DFT-D3¹ with zero-damping, based on the PBE functional, are added in two Ce-to-Lu series: $Ln^{3+}(H_2O)_8$ clusters (part of Fig. 5a of the main text) and Ln^{3+} coordinated to silica surfaces functionalized with two oxalates (Fig. 6c and Fig. 7c of the main text).

Applying D3 to configurations previously optimized using PBE functional yields significant changes in single point total energies in the simulation cells (up to ∼18 eV for the 300+ atom slabs in Fig. 6c of the main text), but minimal changes in the atomic forces. Further optimization of the atomic configurations using PBE-D3 yields only minor differences in the calculated energies, within 0.02-0.12 eV per unit cell, even for the 300+ atom cells.

The trends associated with Ln^{3+} coordinated to 8 water molecules (Fig. 5a of the main text) and Ln^{3+} at an oxalate binding site on silica surfaces (Fig. 6c of the main text) are depicted in Fig. S1a and S1b-c, respectively. The magnitude of $\Delta\Delta E$ generally increases with increasing Ln^{3+} atomic mass; the same behavior is seen with and without D3 corrections.

However, in the middle of the Ln^{3+} series, Gd and Tb now exhibit a noticeable discontinuity in the slope of $\Delta \Delta E$ in water clusters (Fig. S1a); $\Delta \Delta E$ also becomes non-monotonic there. The oxalate ligands exhibit a smaller discontinuity in the slope of $\Delta \Delta E$ at the same position (Fig. S1b). $\Delta\Delta\Delta E$ (Fig. S1d), which involves subtracting Fig. S1a from Fig. S1b, retains this non-monotonic feature at the Gd/Tb location; otherwise it is similar to Fig. 8c of the main text. Qualitatively, the slope going down the Ln series remain steeper on functionalized silica surfaces than with Ln^{3+} coordinated to 8 water molecules, with or without D3. Quantitatively, the $\Delta\Delta E$'s differ by at most 0.25 eV after adding D3 corrections.

The non-monotonicity at the Gd/Tb positions for water clusters (Fig. S1a) may reflect the "the gadolinium break," observed in some properties like ionic radii, formation constants of complexes, and solvent extraction equilibria.² However, non-monotonic trends in hydration free energies have not been previously predicted or observed in experiments,^{4,5} to the best of our knowledge. Instead, Marcus's analysis yields a non-monotonic step at the Pm/Nd position (Fig. S1a; the rise from Yb to Lu there is a possible typo in reported data).⁵ The non-monotonic behavior in Fig. S1a may or may not be an artifact of PBE-D3, and should be the subject of future studies. Finally, a recent work has shown that revPBE-D3 yields more significant disagreement with CCSD(T) predictions than revPBE (no dispersion correction)

FIG. S1: Clockwise: (a) Orange and blue crosses: ∆∆E with/without D3 corrections for Ln³⁺(H₂O)₈; purple circles: $\Delta \Delta E$ for Ln³⁺(H₂O)₉; red line: experimental data.⁵ (b) $\Delta \Delta E$ associated with Ln³⁺ at a binding site where two oxalate ligands are attached to β -crystobalite (110) surfaces, starting from Ce^{3+} -based AIMD configurations. Orange and blue: with/without D3 corrections. (c) $\Delta \Delta E$ associated with Ln³⁺ at a binding site where two oxalate ligands are attached to β -crystobalite (110) surfaces, starting from Lu³⁺-based AIMD configurations Orange and blue: with/without D3 corrections. (d) $\Delta \Delta \Delta E$ for the Ce³⁺-generated configurations (subtracting (b) from (a)). Orange line/red crosses and blue line/green crosses: with/without D3 corrections, respectively.

for interactions between water and some cations. 6 The hydration numbers calculated without the D3 correction are also in better agreement with experiments.

In summary, including D3 corrections does not qualitatively change our conclusions in the main text.

S2. WATER AT T=0 K

This section discusses the optimization of water configurations. We speculate in the main text that adding multiple layers of water beyond the first hydration shell is likely to yield amorphous ice in the limit of a large number of H_2O molecules, despite the fact that a perfect optimization module should yield ice I_h . Such trapping in local minima of the energy landscape is often observed in systems with three-dimensional hydrogen bond networks – even though molecular codes which model isolated molecular system (no periodic boundary conditions, PBC) tend to allow rotation of molecular sub-groups and permits more efficient optimization (Fig. S2a). In contrast, PBC DFT codes generally relax atomic configurations by moving all atoms separately according to forces on each of them, without regard for molecular connectivity (Fig. S2b), and are even less efficient in escaping from metastable states.

To partially demonstrate the propensity for water becoming trapped in glassy metastable configurations, we perform molecular dynamics (MD) simulations of liquid water at 1.0 g/cc density, using 256 SPC/E water molecules⁷ in a cubic periodic cell of dimensions $(19.71 \text{ Å})^3$ and the LAMMPS code.⁸ After equilibration, a $K^+(Br^-)$ complex with a constrained 2.86 Å interionic distance (gas phase optimal distance according to DFT/PBE) replaces two neighboring H_2O . This system, which mimicks the existence of multiple hydration shells around a contact ion pair, is equilibrated for 2 ns at $T=300$ K. At the 1- and 2-ns points, the snapshots are optimized using the conjugate gradient module of the LAMMPS code with all constraints removed. The final configurations are depicted in Figs. S2c-d. They are indeed consistent with the ion pair embedded in amorphous ice. The total energies of the two simulation cells differ by 1.34 eV, which illustrates the variability of energies in amorphous systems.

If the conjugate gradient module can predict the most favorable molecular configuration (including lattice constants), in the limit of a large number of water molecules ("N hydration shells" at large N), it should end up predicting ice Ice I_h, the most stable structure for the SPC/E water model.⁹ First we optimize a simulation cell with 288 SPC/E water molecules in a $15.21 \times 26.34 \times 21.43$ Å³ Ice $1_h 2 \times 4 \times 3$ supercell. Then we replace a neighboring water pair with a K⁺-Br[−] complex and re-optimize. To properly compare with the Fig. S2c-d configurations which have 254 water molecules, we subtract the per- H_2O energy, computed in the pure ice I^h simulation cell, for the 32 excess 32 H2O molecules in Fig. S2e. The corrected energy of this simulation cell is indeed more favorable than the more stable of the amorphous ice configuration quenched from liquid state snapshot by a very significant 17.12 eV.

We conclude that (1) optimizing "multi-layer water" configurations generally yield ions embedded in glassy ice; and (2) embedding the same ionic complex in ice I_h is far more favorable and would have been the most energetically favorable result if the optimization module were able to arrive at I_h geometry spontaneously.

FIG. S2: Schematics of configuration optimization in (a) quantum chemistry codes (left) vs. that in (b) periodic boundary conditions DFT codes (right). Red, white, blue, grey, and green denote O, H, N, C, and Ce atoms. The arrow indicates hindered H2O rotation motion which can break the hydrogen bond shown as a dashed line. (c)-(d) Two optimized K⁺Br⁻/254 H₂O configurations that have different, initial MD-generated starting points. Their relative energies are 0.00 eV and 1.34 eV, respectively. (e) Simulation cell with K+Br[−] in ice Ih, with a relative energy of -17.12 eV after correcting for number of H_2O molecules. Red, white, pink, and green, denote O , H , K , and Br atoms.

S3. HYDRATION NUMBERS

AIMD and DFT predictions of hydration (or coordination) numbers (CN) are discussed in this section. Our assertion is that predicting CN that differ from measurements by ± 1 is almost inevitable, and is expected to lead to small errors in hydration free energy (ΔG_{hvd}) differences.

The schematic in Fig. S3a illustrates the competing factors that determine CN's. Ionwater attraction favors large CN's while water-water repulsion in the first hydration shell favores small CN's. An intermediate CN value optimizes the free energy. In our experience,¹⁰ changing CN from the optimal value by \pm 1 typically increases the free energy by only $\langle 2 \kappa_{\rm B}T$ at room temperature (∼0.05 eV). As DFT functionals do not have "chemical accuracy" (i.e., accuracy within 1 kcal/mol or ∼0.04 eV) compared with experiments, ±1 errors in CN become difficult to avoid. For the same reason, the PBE functional typically predicts a CN which is smaller than that calculated using BLYP.¹⁰ The implicit solvent approximation used also exhibits systematic error. But the corresponding relative error in ΔG_{hyd} is expected to be small.

Experimentally, it is known that the early Ln^{3+} are 9-coordinated and the late Ln^{3+} are 8-coordinated.³ If we assume that $\Delta \Delta E$ s for CN=8 and CN=9 both linearly increase in magnitudes with progression down the series, the two restricted ΔG_{hvd} curves must cross (Fig. S3b). Near the crossing point 8- and 9-coordination must have very similar ΔG_{hyd} .

Next we use the gaussian suite of programs¹¹ to estimate the $CN=8$ and $CN=9$ curves using the PBE functional and two implicit solvent approximations: $PCM¹²$ (Fig. S3c) and SMD (Fig. S3d).¹³ The SDD basis set is adopted.¹⁴ Only La^{3+} and Lu^{3+} hydration free energies are explicitly calculated; the empty and full 4f shells of these end-point elements make the calculations much easier than for the Ln^{3+} in between. The CN=9 configuration is taken to be the reference energy for each Ln^{3+} , while the hydration free energy of CN=8 is

$$
\Delta G_{\text{CN=8}} = \Delta G_{\text{CN=9}} - \Delta G_{\text{water}}.\tag{S1}
$$

An entropic correction of 0.186 eV is added to convert the gas phase translational entropy reported in gaussian for gaseous water at 1.0 atm. pressure, to that appropriate to liquid water density.

In the PCM calculations, after interpolating ΔG_{hyd} for both CN=8 and CN=9 as straight

FIG. S3: (a) Schematic of competing factors that determine CN's. (b) Schematic of experimental $\Delta\Delta G_{\text{hyd}}$ at fixed CN=8 and CN=9. (c)-(d) Interpolated $\Delta\Delta G_{\text{hyd}}$ at fixed CN=8 and CN=9 for the PCM and SMD implicit solvation methods. The crosses and dashed lines represent all H_2O in the first hydration shell (both $CN=8$ and $CN=9$). The empty circles are for 9 H₂O molecules in the simulation cell, but with one H_2O in the second hydration shell.

lines, it is estimated that the entire Ln^{3+} series should have $CN=9$ (Fig. S3c). The SMD results appear to exhibit cross-over behavior, with 9-coordination favored for La^{3+} and 8coordination favored for Lu³⁺, ensuring that, for some intermediate Ln³⁺, the ΔG_{hyd} for CN=8 and CN=9 must be similar (Fig. S3d).

But in fact, the situation is more complex. Our CN=9 coordination are initiated using $U(H_2O)$ ₉ configurations,¹⁵ and optimized via the gaussian quasi-Newton module. If we use the conjugate-gradient method for optimization, in at least one case an H_2O molecule is spontaneously ejected from the first hydration shell, forming hydrogen bonds with other first hydration shell H₂O molecules. Restarting from this latter configuration, the ΔG_{hyd} , now for a 9-water cluster but in reality a $CN=8$ configuration, proves more favorable (circles in Fig. S3c-d) than both the inner sphere CN=9 and the CN=8-plus-free-water. This is true of both La^{3+} and Lu^{3+} , and of both PCM and SMD implicit solvent methods. Intepolating from the end-point elements, we argue $CN=8$ is favored for all Ln^{3+} for the PBE functional.

The above describes the results of static DFT calculations with implicit solvent approximations. The CN's in a number of dynamical AIMD simulations of La^{3+} in water has also been reported (Ref. 4, Table 1). Both CN=8 and CN=9 have been predicted. The EXAFS value is 9.1.¹⁶ We have not considered La^{3+} in our AIMD/PBE simulations, but judging from our CN=7.3 and 7.4 for Dy^{3+} and Tb^{3+} in water (Table 1 of the main text)– both less than $CN=8$ – we expect the $AIMD/PBE$ CN to be slightly underestimated compared with the accepted experimental values of 8-9. In the literature, the combination of B3LYP and PCM have been shown to yield Ln^{3+} CN's which cross over from 9 to 8,³ in agreement with experiments. However, hybrid DFT functionals like B3LYP would have been prohibitively expensive to use for the DFT and AIMD simulation cells with 300+ atoms herein. Instead we have chosen the arguably less accurate PBE functional for our large scale simulations.

In conclusion, due to the finite accuracy of DFT functioals, the hydration or coordination numbers (CN) predicted for Ln^{3+} can easily deviate from the experimental value by ± 1 .

S4. PAIR CORRELATION FUNCTIONS AND OTHER DETAILS

Fig. S4a-h depict the pair correlation functions $(g(r))$ between Dy^{3+} or Tb³⁺ and the oxygen sites of H_2O and ligands described in Table 1 of the main text. Integrating the first peak yields the hydration numbers listed in Table 1 of the main text.

Table S1 describes the trajectory lengths and number of water molecules associated with various simulation cells associated with Fig. 1 of the main text. In AIMD simulations with only water and Ln^{3+} , the cell dimensions are $12.42\times12.42\times12.42$ Å³ with 64 H₂O molecules; such simulation cells have $+3|e|$ net charges, where |e| is the electronic charge. AIMD simulation cells with ligands are charge-neutral, have $18\times12\times12$ Å³ dimensions, and 85 H2O molecules. AIMD simulation cells used in potential-of-mean-force simulations (Fig. 2 of the main text) are charge-neutral, have $24\times12\times12$ Å³ dimensions, and 111 H₂O molecules.

We have performed additional calculations to compare the AIMD $\Delta\Delta G$ calculations used for Fig. 1 and Fig. 3 of the main text, and the $\Delta\Delta E$ method associated with Fig.6-Fig. 8 of the main text. Fig. S5 depicts an optimized cluster taken from the trajectory associated with Fig. 1b of the main text. Most H_2O molecules except those coordinated to the Dy^{3+} and to the two sulfate anions are removed, and atomic optimization is carried out in "vacuum;" this procedure is similar to the Figs. 6-8 calculations of the main text. The lanthanide cation is 6-coordinated in Fig. S5. $\Delta\Delta E$ for switching between Tb³⁺ and Dy³⁺ for is -0.33 eV, very similar to the $\Delta\Delta G$ calculated using AIMD perturbation with the entire simulation cell filed with water (-0.30 eV). This illustrates the internal consistency of the two methods.

FIG. S4: $g(r)$ between Ln³⁺ and oxygen atoms in both water and ligands. (a)-(d): Dy³⁺ in pure water, coordinated with $\text{HPO}_4^{2-}/\text{H}_2\text{PO}_4^-$, with $\text{SO}_4^{2-}/\text{H}_2\text{PO}_4^-$, and with $\text{SO}_4^{2-}/\text{HSO}_4^-$, respectively. (e)-(h): same as (a)-(d) but with Tb^{3+} rather than Dy^{3+} .

FIG. S5: Cluster configuration taken from an AIMD snapshot associated with the trajectory depicted in Fig. 1b of the main text, with a SO_4^{2-} and a SO_4^{2-} H anion coordinated to Dy^{3+} . Red, white, yellow, and green denote O, H, S, and Dy atoms.

system	time system	time
Dy^{3+} in water	263.8 Tb ³⁺ in water	257.3
$\mathrm{Dy^{3+}(PO_4^{3-})_2}$	$54.5 Tb^{3+}(PO^{3-}_4)_2$	67.4
$\mathrm{Dy}^{3+}(\mathrm{SO}_4^{2-})_2$	$74.0 \text{ } \text{Tb}^{3+}(\text{SO}_4^{2-})_2 $	76.6
$\mathrm{Dy}^{3+}(\mathrm{SO}_4^{2-})/\mathrm{PO}_4^{3-}$	$83.4 \text{ } \text{Tb}^{3+}(\text{SO}_4^{2-})/\text{PO}_4^{3-}$	62.5
Dy^{3+} in UiO-66	18.8Tb^{3+} in UiO-66	19.6
PMF (none)	33.2 PMF 2.80	27.2
PMF 2.95	38.7 PMF 3.25	11.7
PMF 3.50	17.2 PMF 3.80	24.1
PMF 4.15	16.4 PMF 4.50	22.5
PMF 4.85	36.6 PMF 5.15	22.8
PMF 5.50	23.3 PMF 5.85	18.3
PMF 6.20	22.5 PMF 6.55	16.4
PMF 6.90	16.7 PMF 7.55	16.7

TABLE S1: AIMD trajectory lengths in picoseconds for different systems. "PMF" refers to potential-of-mean-force simulations of the dissociation of $C_2H_5NH_2$ from Lu^{3+} ; the number is the constraint distance R_o in the constraint potential $V(R) = A(R - R_o)^2$, in units of Angstrom; A is 2 eV/ \AA ². Extra protons are added to SO_4^{2-} and PO_4^{3-} to ensure charge neutrality. The exceptions are simulation cells with only Ln^{3+} and water, which have net $+3|e|$ charges.

S5. FINAL ENERGIES AND CONFIGURATIONS

The $\langle \Delta E_b \rangle_a$ and $\langle \Delta E_a \rangle_a$ described in Eq. 1 of the main text and utilized in Fig. 1 and Fig. 3 of the main text are listed in Table S2. Note that Eq. 1 yields small $\Delta\Delta G$ from substracting large numbers. This is justified because the $\langle \Delta E_b \rangle_a$ and $\langle \Delta E_a \rangle_a$ are correlated and are generated using the configurations along the same AIMD trajectory with pseudopotential "a". The correlated sampling greatly reduces the statistical uncertainty.

	Fig. $1a$	Fig. $1b$	Fig. 1c	Fig. 1d	Fig. 3
			$(\Delta E_{\rm Dv}$ _{Dv} -967.4183 -1344.9576 -1360.7031 -1353.2604 -5316.1160		
			$(\Delta E_{\rm Tb})_{\rm Dv}$ -967.1237 - 1344.6452 - 1360.3903 - 1352.9453 - 5315.8053		
			$(\Delta E_{\rm Dv}$ _{Tb} -967.6740 -1344.8968 -1360.7322 -1352.8316 -5316.4056		
			$(\Delta E_{\rm Tb} _{\rm 7b} _{\rm -967.3918} _{\rm -1344.6054} _{\rm -1360.4337} _{\rm -1352.5394} _{\rm -5316.1042}$		

TABLE S2: $\langle \Delta E_b \rangle_a$ associated with Fig. 1 and Fig. 3 of the main text, in eV. a and b are alternatively Dy and Tb. The energies of the bare Dy^{3+} and Tb^{3+} are subtracted.

Ln^{3+}		8 H ₂ O 7 H ₂ O / OH ⁻ 7 H ₂ O / PYC		9 H ₂ O
	Ce -109.3056	-112.0641	-174.1694 -124.4979	
	$Pr[-109.2928]$	-112.1041	-174.1572 -124.4751	
	Nd -109.2714	-112.1225	-174.1318 -124.4387	
	$Pm - 109.2574$	-112.1483	-174.1096 -124.4041	
	Sm -109.1943	-112.1058	-174.0384 -124.3238	
	Eu ^{-109.1895}	-112.1262	-174.0188 -124.2965	
	Gd -109.1822	-112.1550	-173.9953 -124.2648	
	Tb-109.1445	-112.1417	-173.9498 -124.2054	
	Dy -109.0987	-112.1185	-173.8968 -124.1387	
	Ho -109.0508	-112.0910	-173.8401 -124.0682	
	$Er[-109.0135]$	-112.0645	-173.7955 -124.0132	
	Tm -108.9926	-112.0580	-173.7644 -123.9709	
	Yb -108.9327	-112.0580	-173.7118 -123.8909	
	Lu -108.8780	-111.9939	-173.6612 -123.8249	

TABLE S3: E_{tot} associated with Fig. 4 of the main text, for Ln³⁺(H₂O)₈, Ln³⁺(H₂O)₇(OH⁻), $\text{Ln}^{3+}(\text{H}_2\text{O})_7(\text{PYC})$, and $\text{Ln}^{3+}(\text{H}_2\text{O})_9$.

The optimized energies in simulation cells $(E_{\text{tot}}, Eq. 2)$ of the text) associated with Figs. 4a-c, Fig. 6a-f, and Fig. 9b are listed in Tables S3-S7. Individually E_{tot} are meaningless and cannot be reproduced by other DFT codes. Only the difference of E_{tot} for different Ln^{3+} in the same environment, which becomes Eq. 3 of the main text via Eq. 2 of the main text, are physical quantities.

	Ln^{3+} Fig. 6a Ce Fig. 6a Lu Fig. 6b Ce Fig. 6b Lu	
	Cel-2324.6132 - 2324.5378 - 2340.8677 - 2369.8888	
	Pr -2324.6901 -2324.6335 -2340.9295 -2369.9639	
	Nd -2324.7414 -2324.7036 -2340.9672 -2370.0145	
	Pm -2324.7967 -2324.7817 -2341.0142 -2370.0708	
	Sm -2324.7758 -2324.7790 -2340.9987 -2370.0574	
	Eu -2324.8236 -2324.8541 -2341.0395 -2370.1067	
	Gd -2324.8678 -2324.9244 -2341.0840 -2370.1549	
	Tb -2324.8743 -2324.9508 -2341.0923 -2370.1679	
	Dy -2324.8701 -2324.9671 -2341.0915 -2370.1706	
	Hol-2324.8641 -2324.9828 -2341.0863 -2370.1702	
	Er -2324.8576 -2324.9929 -2341.0799 -2370.1672	
	Tm -2324.8697 -2325.0334 -2341.0961 -2370.1898	
	Yb -2324.8586 -2325.0516 -2341.0908 -2370.1978	
	Lu -2324.8395 -2325.0557 -2341.0716 -2370.1881	

TABLE S4: E_{tot} associated with Fig. 6a-b of the main text. "Ce" and "Lu" refer to configurations started with Ce-based and Lu-based AIMD runs. Recall that Ce- and Lu-configurations can have different number of water molecules, and their energies cannot be directly compared.

The optimized atomic configurations associated with Fig. 4, Fig. 6-8, and Fig. 9b of the main text are found in a separate file (finalconfig.zip). In Fig. 4 of the main text, the lattice dimensions are $16 \times 16 \times 16$ Å³. For systems involving silica substrates (Fig. 6-8, Fig. 9b), the lattice dimensions are $20.25 \times 14.32 \times 36.00$ Å³. Configurations for each of the 14 Ln³⁺ for each of the 6 silica binding sites associated with Fig. 6 of the main text are listed twice – once for the Ce-initiated AIMD configurations, and once for the Lu-initiated configurations. Note that these starting points can lead to different number of H_2O molecules present.

The AIMD simulations associated with Figs. 1-3 generate many gigabytes of data. These will be available upon request to the authors.

Ln^{3+}				Fig. 6c Ce Fig. 6c Lu Fig. 6d Ce Fig. 6d Lu
			Cel-2234.6401 -2234.6068 -2259.4795 -2262.4338	
			$Pr[-2234.7081] - 2234.6905] - 2259.5494] - 2262.5264$	
			Nd -2234.7652 -2234.7510 -2259.5974 -2262.5907	
			Pm -2234.8345 -2234.8205 -2259.6450 -2262.6598	
			Sm -2234.8328 -2234.8182 -2259.6236 -2262.6547	
			Eu -2234.8903 -2234.8788 -2259.6620 -2262.7105	
			Gd -2234.9545 -2234.9465 -2259.7173 -2262.7735	
			Tb -2234.9812 -2234.9733 -2259.7296 -2262.8034	
			Dy -2234.9990 -2234.9928 -2259.7326 -2262.8179	
			Ho \mid -2235.0153 \mid -2235.0082 \mid -2259.7287 \mid -2262.8310	
			$Er[-2235.0314]-2235.0232]-2259.7223]-2262.8358$	
			Tm -2235.0680 -2235.0665 -2259.7363 -2262.8692	
			Yb -2235.0878 -2235.0901 -2259.7426 -2262.8886	
			Lu -2235.0984 -2235.1017 -2259.7320 -2262.8912	

TABLE S5: E_{tot} associated with Fig. 6c-d of the main text. "Ce" and "Lu" refer to configurations started with Ce-based and Lu-based AIMD runs. Recall that Ce- and Lu-configurations can have different number of water molecules, and their energies cannot be directly compared.

Ln^{3+}		Fig. 6e Ce Fig. 6e Lu Fig. 6f Ce Fig. 6f Lu	
	Ce -2574.6407 -2533.5817 -2172.8261 -2172.5092		
	Pr -2574.6522 -2533.6697 -2172.9017 -2172.6040		
	Nd -2574.7033 -2533.7350 -2172.9550 -2172.6756		
	Pm - 2574.7672 - 2533.8028 - 2173.0143 - 2172.7597		
	Sm -2574.7628 -2533.7966 -2173.0016 -2172.7605		
	Eu -2574.8244 -2533.8559 -2173.0530 -2172.8291		
	Gd -2574.8753 -2533.9137 -2173.1167 -2172.9126		
	Tb -2574.8892 -2533.9393 -2173.1349 -2172.9531		
	Dy -2574.9361 -2533.9553 -2173.1451 -2172.9800		
	Ho -2574.9480 -2533.9742 -2173.1168 -2173.0039		
	$Er[-2574.9458]-2533.9852]-2173.1617[-2173.0212$		
	Tm -2574.9912 -2534.0328 -2173.2006 -2173.0670		
	Yb -2575.0065 -2534.0529 -2173.2163 -2173.1050		
	Lu -2575.0222 -2534.0558 -2173.2163 -2173.1254		

TABLE S6: E_{tot} associated with Fig. 5c-d of the main text. "Ce" and "Lu" refer to configurations started with Ce-based and Lu-based AIMD runs. Recall that Ce- and Lu-configurations can have different number of water molecules, and their energies cannot be directly compared.

	Ce -2264.7720 Pr -2264.8630 Nd -2264.9381	
	$\rm{ Pm }\text{-}2265.0212\,{\rm{ Sm }\text{-}2265.0251}$ Eu -2265.0922	
		Gd -2265.1687 Tb -2265.1981 Dy -2265.2185
		Ho -2265.2385 Er -2265.2510 Tm -2265.2925
	Yb -2265.2990 Lu -2265.3234	

TABLE S7: $E_{\rm tot},$ in eV, associated with Fig. 9b of the main text.

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