

## Supporting Information

### First-principles study of electronic structure and point defects in higher manganese silicide $\text{Mn}_4\text{Si}_7$

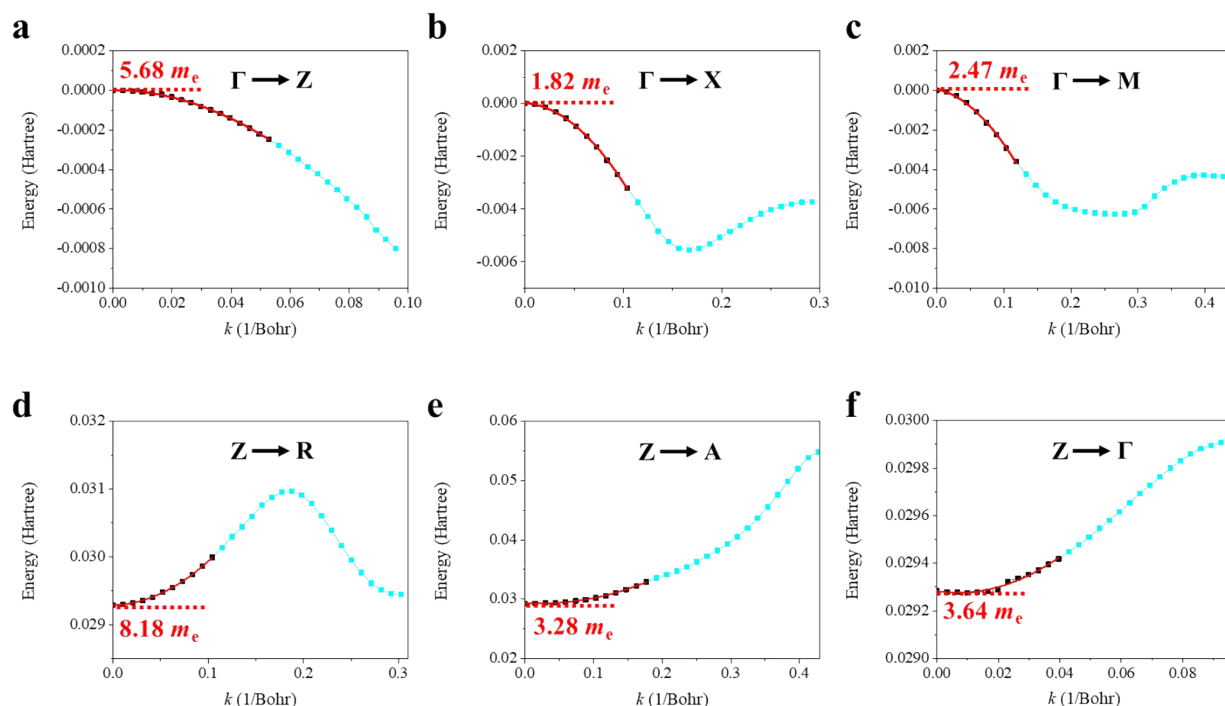
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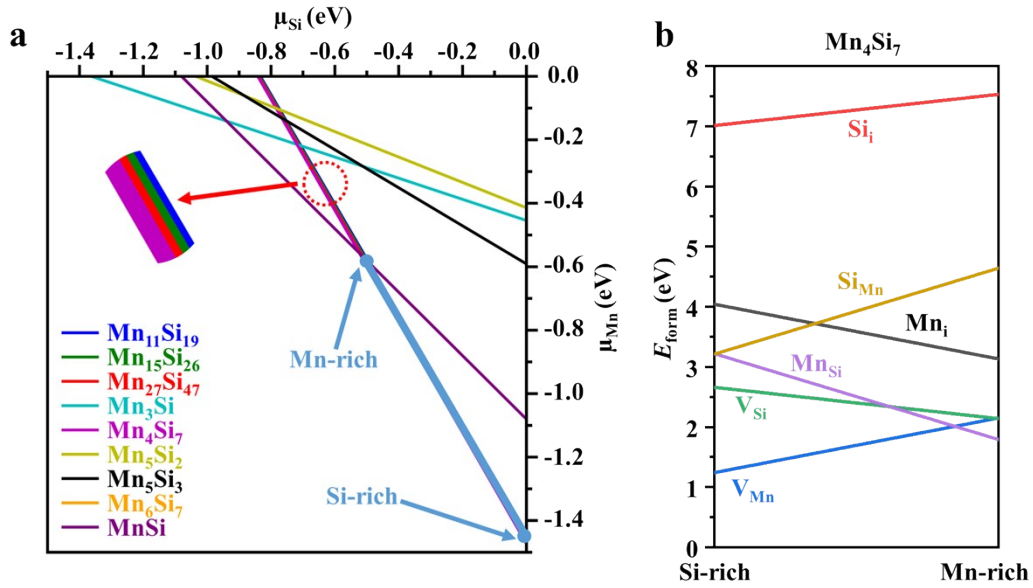
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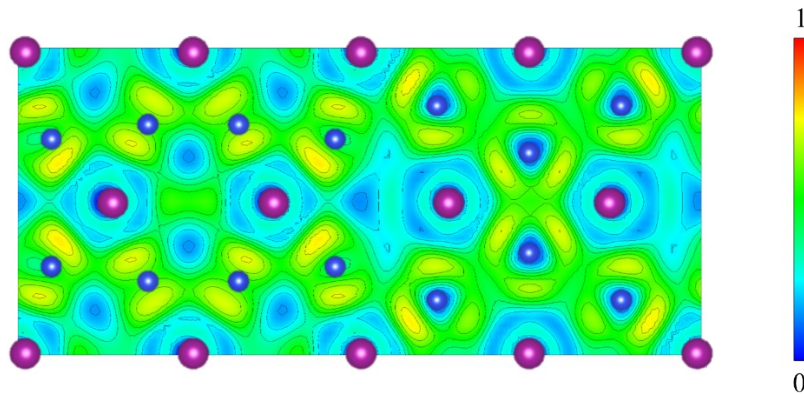


**Fig. S1** The effective masses of hole and electron carriers at VBM (a, b, c) and CBM (d, e, f) for  $\text{Mn}_4\text{Si}_7$  were evaluated along different directions. The black points near VBM and CBM are fitted using a quadratic term  $y =$

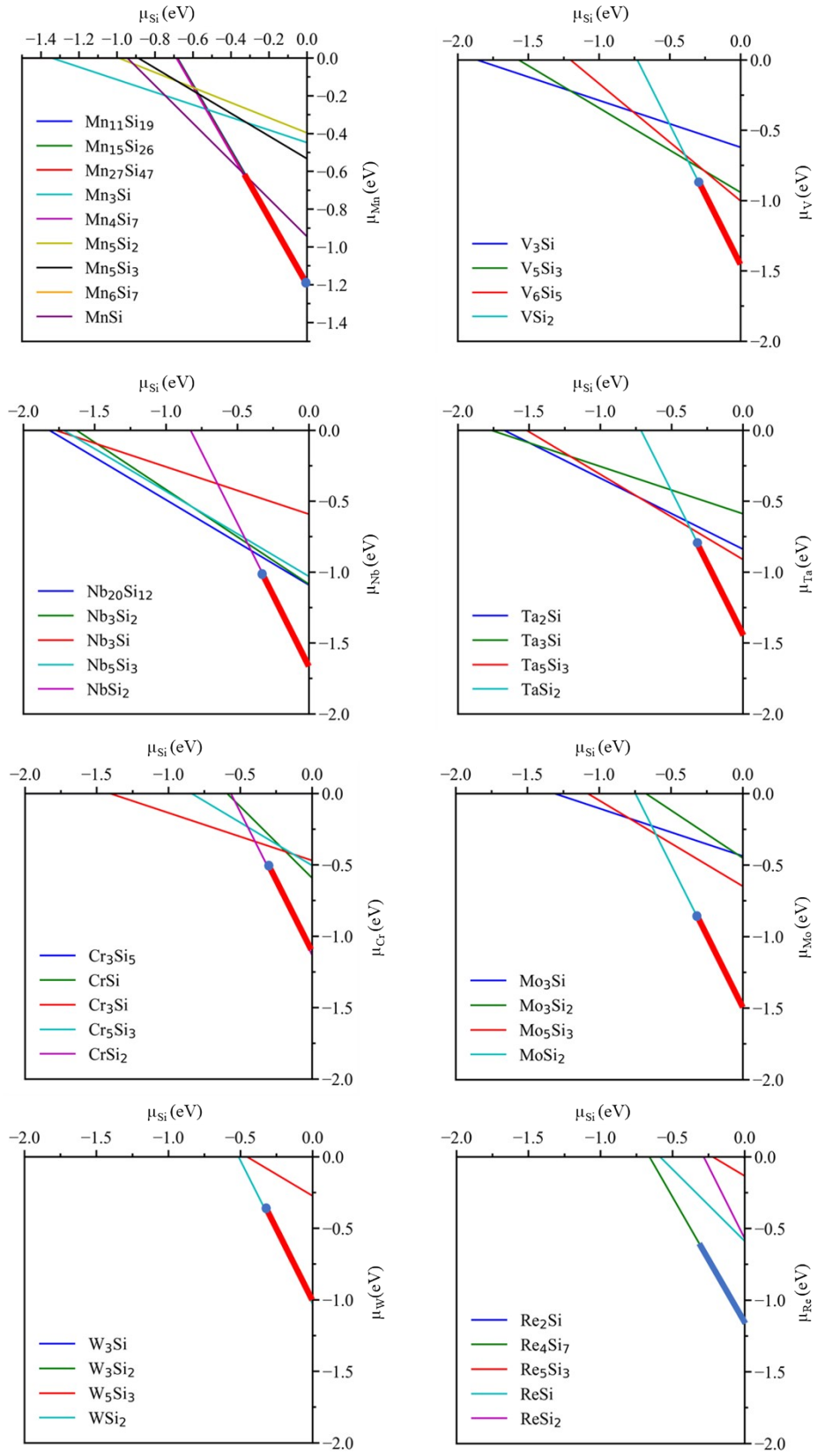
$A+Bx+Cx^2$ . The red line represents the fitting result, where the effective mass is given by  $\frac{1}{2C}m_e$ .

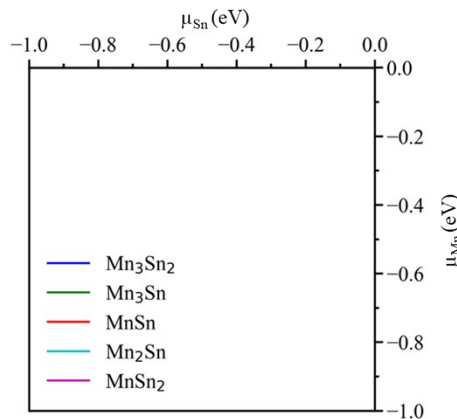
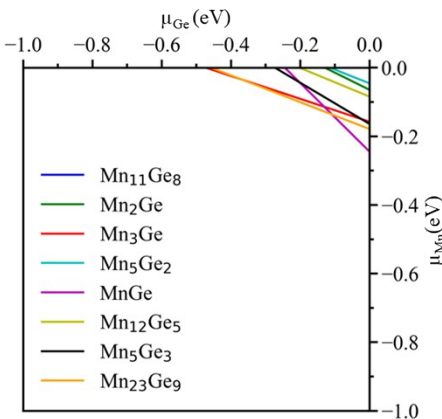
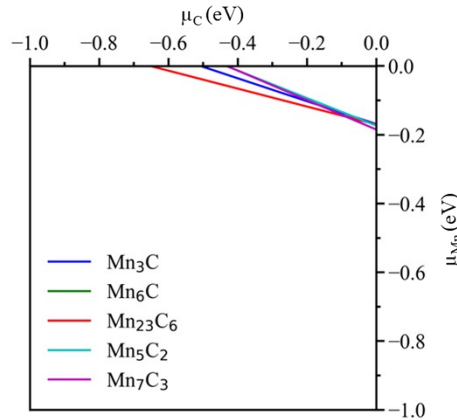
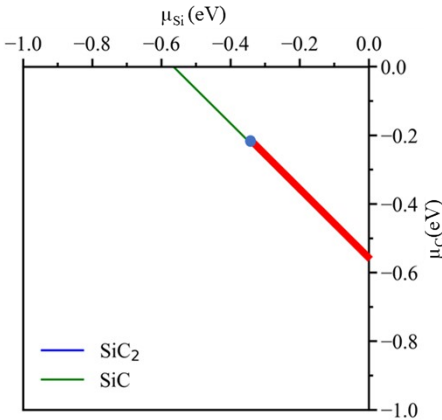
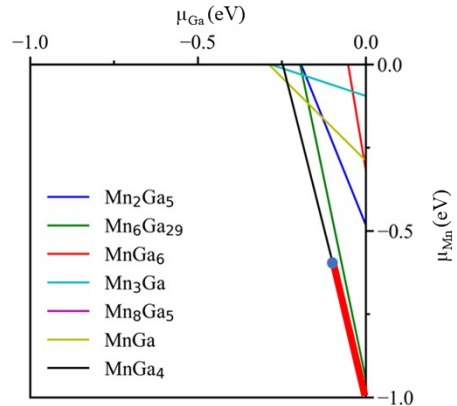
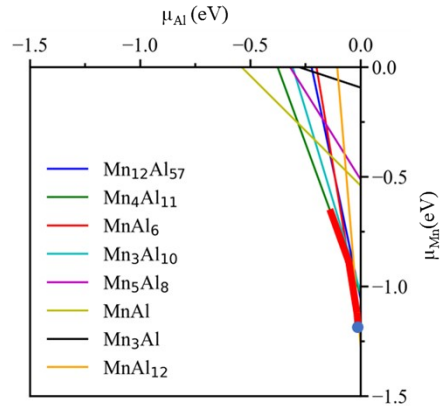
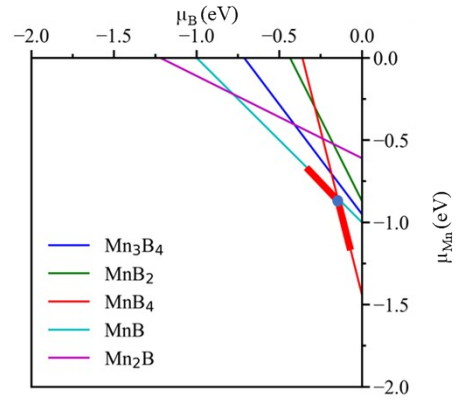
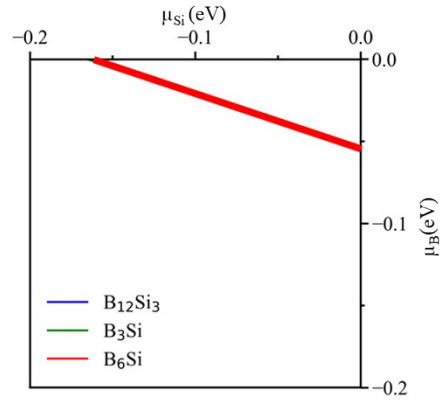


**Fig. S2** (a) The calculated stable region of  $\text{Mn}_4\text{Si}_7$  in the chemical potential space spanned by  $\mu_{\text{Mn}}$  and  $\mu_{\text{Si}}$ . Each straight line corresponds to  $x \mu_{\text{Mn}} + y \mu_{\text{Si}} = \Delta H(\text{Mn}_x\text{Si}_y)$ . The allowed region is delineated by a bold blue line. The inset, magnifying the area within the red circle, illustrates the constituted compounds of HMS, namely  $\text{Mn}_4\text{Si}_7$ ,  $\text{Mn}_{11}\text{Si}_{19}$ ,  $\text{Mn}_{15}\text{Si}_{26}$  and  $\text{Mn}_{27}\text{Si}_{47}$ . (b) The formation energies of the intrinsic point defects in  $\text{Mn}_4\text{Si}_7$  as a function of chemical potentials between the Si-rich and Mn-rich conditions. These results are obtained using the PBEsol functional.



**Fig. S3** Two-dimensional Electron Localization Function (ELF) contour plots on (110) plane of  $\text{Mn}_4\text{Si}_7$  in Fig. 1. Mn and Si atoms are represented by purple and blue spheres. The contour lines are spaced at intervals of 0.08. The closer the ELF is to zero, the more ionic the bonding features.





**Fig. S4** Determination of the allowed region for chemical potentials. Each straight line corresponds to  $x \mu_{\text{Mn}} + y \mu_{\text{Si}} = \Delta H(\text{Mn}_x\text{Si}_y)$ ,  $x \mu_D + y \mu_{\text{Si}} = \Delta H(D_x\text{Si}_y)$ , and  $x \mu_{\text{Mn}} + y \mu_D = \Delta H(\text{Mn}_x D_y)$ .  $\text{Mn}_x\text{Si}_y$ ,  $D_x\text{Si}_y$ , and  $\text{Mn}_x D_y$  represent the Mn-Si binary compounds, D-Si binary compounds, and Mn-D binary compounds, respectively.  $\Delta H$  is the calculated enthalpy of formation. The thick solid red lines mark the allowed chemical potential regions and the blue dots indicate the condition used for obtaining Fig. 5, at which the corresponding formation energy is the lowest. It is worth noting that the straight lines for binary compounds with  $\Delta H > 0$  will not be displayed.

**Table S1** Total magnetization for  $\text{Mn}_4\text{Si}_7$  containing intrinsic defects.

Defect type	Total magnetization ( $\mu_B$ )
$V_{\text{Mn}}$ ( $\text{Mn}_{63}\text{Si}_{112}$ )	0.99
$V_{\text{Si}}$ ( $\text{Mn}_{64}\text{Si}_{111}$ )	3.71
$\text{Mn}_i$ ( $\text{Mn}_{65}\text{Si}_{112}$ )	3.03
$\text{Si}_i$ ( $\text{Mn}_{64}\text{Si}_{113}$ )	4.00
$\text{Mn}_{\text{Si}}$ ( $\text{Mn}_{65}\text{Si}_{111}$ )	3.01
$\text{Si}_{\text{Mn}}$ ( $\text{Mn}_{63}\text{Si}_{113}$ )	0.59

**Table S2** The charge state of Mn and Si for Mn<sub>4</sub>Si<sub>7</sub> obtained by using Bader charge analysis. The lattice constants are  $a = b = 5.498 \text{ \AA}$  and  $c = 17.35 \text{ \AA}$ .

Atom	$x$	$y$	$z$	Charge state
Mn1	0.000000	0.000000	0.250000	0.12
Mn2	0.000000	0.000000	0.750000	-0.01
Mn3	0.000000	0.000000	0.000000	-0.21
Mn4	0.000000	0.000000	0.500000	-0.21
Mn5	0.500000	0.500000	0.130090	-0.19
Mn6	0.500000	0.500000	0.869910	-0.21
Mn7	0.500000	0.500000	0.630090	-0.21
Mn8	0.500000	0.500000	0.369910	-0.19
Mn9	0.000000	0.500000	0.065081	-0.11
Mn10	0.500000	0.000000	0.934919	-0.10
Mn11	0.000000	0.500000	0.565081	-0.10
Mn12	0.500000	0.000000	0.434919	-0.11
Mn13	0.000000	0.500000	0.308212	-0.05
Mn14	0.500000	0.000000	0.691788	-0.10
Mn15	0.000000	0.500000	0.808212	-0.10
Mn16	0.500000	0.000000	0.191788	-0.05
Si1	0.345942	0.769431	0.038804	0.07
Si2	0.654058	0.230569	0.038804	0.14
Si3	0.769431	0.654058	0.961196	0.09
Si4	0.230569	0.345942	0.961196	0.10
Si5	0.345942	0.230569	0.538804	0.05
Si6	0.654058	0.769431	0.538804	0.14
Si7	0.769431	0.345942	0.461196	0.11
Si8	0.230569	0.654058	0.461196	0.11
Si9	0.199407	0.155412	0.112824	-0.03
Si10	0.800592	0.844588	0.112824	0.04
Si11	0.155412	0.800592	0.887176	0.05
Si12	0.844588	0.199407	0.887176	0.03

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Si13	0.199407	0.844588	0.612824	0.02
Si14	0.800592	0.155412	0.612824	0.05
Si15	0.155412	0.199407	0.387176	0.02
Si16	0.844588	0.800592	0.387176	-0.01
Si17	0.158878	0.678921	0.182512	0.05
Si18	0.841122	0.321079	0.182512	-0.03
Si19	0.678921	0.841122	0.817488	-0.01
Si20	0.321079	0.158878	0.817488	0.08
Si21	0.158878	0.321079	0.682512	0.07
Si22	0.841122	0.678921	0.682512	0.00
Si23	0.678921	0.158878	0.317488	0.04
Si24	0.321079	0.841122	0.317488	-0.03
Si25	0.333920	0.333920	0.250000	0.08
Si26	0.666080	0.666080	0.250000	0.25
Si27	0.333920	0.666080	0.750000	0.10
Si28	0.666080	0.333920	0.750000	0.23

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**Table S3** Binary compounds of Mn-Si-D systems which are used in determining the chemical potentials for calculating defect formation energy. All structures are obtained from ICSD.

<b>Mn-Si</b>	Mn <sub>11</sub> Si <sub>19</sub> (P4 <sub>1</sub> cd #110), Mn <sub>15</sub> Si <sub>26</sub> (I-4 <sub>2</sub> d #122), Mn <sub>27</sub> Si <sub>47</sub> (P4 <sub>1</sub> cd #110), Mn <sub>3</sub> Si (Fm-3m #225), Mn <sub>4</sub> Si <sub>7</sub> (P-4c2 #116), Mn <sub>5</sub> Si <sub>2</sub> (P4 <sub>2</sub> 1 <sub>2</sub> #92), Mn <sub>5</sub> Si <sub>3</sub> (P2 <sub>1</sub> /m #11, Cmcmm #63, P6 <sub>3</sub> /mcm #193), Mn <sub>6</sub> Si <sub>7</sub> (R-3m #166), MnSi (P2 <sub>1</sub> 3 #198)
<b>V-Si</b> <b>Mn-V</b>	V <sub>3</sub> Si (Pm-3n #223), V <sub>5</sub> Si <sub>3</sub> (P6 <sub>3</sub> /mcm #193, I4/mcm #140), V <sub>6</sub> Si <sub>5</sub> (Immm #71, Ibam #72), VSi <sub>2</sub> (P6 <sub>2</sub> 22 #180)
<b>Nb-Si</b> <b>Mn-Nb</b>	Nb <sub>3</sub> Si <sub>2</sub> (P4/mbm #127), Nb <sub>3</sub> Si (Pm-3n #223, Pm-3m #221, P4 <sub>2</sub> /n #86), Nb <sub>5</sub> Si <sub>3</sub> (I4/mcm #140, P6 <sub>3</sub> /mcm #193), NbSi <sub>2</sub> (P6 <sub>2</sub> 22 #180)
<b>Ta-Si</b> <b>Mn-Ta</b>	Ta <sub>2</sub> Si (I4/mcm #140), Ta <sub>3</sub> Si (P4 <sub>2</sub> /n #86), Ta <sub>5</sub> Si <sub>3</sub> (P6 <sub>3</sub> /mcm #193, I4/mcm #140), TaSi <sub>2</sub> (P6 <sub>2</sub> 22 #180)
<b>Cr-Si</b> <b>Mn-Cr</b>	Cr <sub>3</sub> Si (Pm-3n #223), Cr <sub>3</sub> Si <sub>5</sub> (I4/mcm #140), Cr <sub>5</sub> Si <sub>3</sub> (I4/mcm #140), CrSi (P2 <sub>1</sub> 3 #198), CrSi <sub>2</sub> (P6 <sub>2</sub> 22 #180, I4/mmm#139)
<b>Mo-Si</b> <b>Mn-Mo</b>	Mo <sub>3</sub> Si (Pm-3n #223), Mo <sub>3</sub> Si <sub>2</sub> (P4/mbm #127), Mo <sub>5</sub> Si <sub>3</sub> (I4/mcm #140, P6 <sub>3</sub> /mcm #193), MoSi <sub>2</sub> (P6 <sub>2</sub> 22 #180, I4/mmm#139)
<b>W-Si</b> <b>Mn-W</b>	W <sub>3</sub> Si (Pm-3n #223) W <sub>3</sub> Si <sub>2</sub> (P4/mbm #127), W <sub>5</sub> Si <sub>3</sub> (I4/mcm #140), WSi <sub>2</sub> (P6 <sub>2</sub> 22 #180, I4/mmm#139)
<b>Re-Si</b> <b>Mn-Re</b>	Re <sub>2</sub> Si (P2 <sub>1</sub> /c #14), Re <sub>4</sub> Si <sub>7</sub> (Cm #8), Re <sub>5</sub> Si <sub>3</sub> (I4/mcm #140), ReSi (P2 <sub>1</sub> 3 #198), ReSi <sub>2</sub> (I4/mmm #139, Immm #71)
<b>B-Si</b> <b>Mn-B</b>	B <sub>12</sub> Si <sub>3</sub> (R-3m #166), B <sub>3</sub> Si (Imma #74), B <sub>6</sub> Si (Pm-3m #221) MnB (Pnma #62, P4 <sub>2</sub> /ncm, #138 ), Mn <sub>2</sub> B (I4/mcm #140), MnB <sub>2</sub> (P6/mmm #191), MnB <sub>4</sub> (Pnmm #58, C2/m #12, P2 <sub>1</sub> /c #14), Mn <sub>3</sub> B <sub>4</sub> (Immm #71)



<b>Al-Si</b>	Al <sub>8</sub> Si <sub>38</sub> (P1 #1)
<b>Mn-Al</b>	MnAl (P4/mmm #123), Mn <sub>3</sub> Al (F-43m #216), Mn <sub>3</sub> Al <sub>10</sub> (P6 <sub>3</sub> /mmc #194), Mn <sub>4</sub> Al <sub>11</sub> (P-1 #2), MnAl <sub>12</sub> (Im-3 #204), Mn <sub>12</sub> Al <sub>57</sub> (Pm-3 #200), MnAl <sub>6</sub> (Cmcm #63), Mn <sub>5</sub> Al <sub>8</sub> (R3m #160)
<b>Ga-Si</b>	MnGa (P4/mmm #123, R-3m #166), Mn <sub>3</sub> Ga (Fm-3m #225, P6 <sub>3</sub> /mmc #194), Mn <sub>6</sub> Ga <sub>29</sub> (P-1 #2),
<b>Mn-Ga</b>	MnGa <sub>4</sub> (Im-3m #229), Mn <sub>2</sub> Ga <sub>5</sub> (P4/mbm #127), Mn <sub>8</sub> Ga <sub>5</sub> (I-43m #127), MnGa <sub>6</sub> (Cmcm #63)
<b>In-Si</b>	None
<b>Mn-In</b>	
<b>C-Si</b>	SiC (F-43m #216, Fm-3m #225, P3m1 #156, P6 <sub>3</sub> /mc #186, R3m #160), SiC <sub>2</sub> (P4 <sub>2</sub> /mmc #131, Pa-3 #205)
<b>Mn-C</b>	Mn <sub>3</sub> C (Pnma #62), Mn <sub>6</sub> C (Fd-3m #227), Mn <sub>11</sub> C <sub>3</sub> (F23 #196), Mn <sub>5</sub> C <sub>2</sub> (C2/c #15), Mn <sub>7</sub> C <sub>3</sub> (Pnma #62), Mn <sub>23</sub> C <sub>6</sub> (Fm-3m #225)
<b>Ge-Si</b>	MnGe (P2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub> #19, P2 <sub>1</sub> 3 #198), Mn <sub>2</sub> Ge (F-43m #216, P6 <sub>3</sub> /mmc #194), Mn <sub>3</sub> Ge (Pm-3m #221, Fm-3m #225, P6 <sub>3</sub> /mmc #194), Mn <sub>5</sub> Ge <sub>2</sub> (Ibam #72, P3c1 #158), Mn <sub>5</sub> Ge <sub>3</sub> (P6 <sub>3</sub> /mcm #193),
<b>Mn-Ge</b>	Mn <sub>12</sub> Ge <sub>5</sub> (R32 #155), Mn <sub>11</sub> Ge <sub>8</sub> (Pnma #62), Mn <sub>23</sub> Ge <sub>9</sub> (P3c1 #158)
<b>Sn-Si</b>	MnSn (Pm-3m #221, Fm-3m #225, P6 <sub>3</sub> /mmc #194, F-43m #216), MnSn <sub>2</sub> (I4/mcm #140), Mn <sub>2</sub> Sn
<b>Mn-Sn</b>	(P6 <sub>3</sub> /mmc #194), Mn <sub>3</sub> Sn (P6 <sub>3</sub> /mmc #194, Fm-3m #225), Mn <sub>3</sub> Sn <sub>2</sub> (Pnma #62)

**Table S4** Total magnetization for  $\text{Mn}_4\text{Si}_7$  containing extrinsic defects.

Dopant type	Total magnetization ( $\mu_B$ )
None ( $\text{Mn}_{16}\text{Si}_{28}$ )	0
V ( $\text{Mn}_{15}\text{VSi}_{28}$ )	1.73
Nb ( $\text{Mn}_{15}\text{NbSi}_{28}$ )	0.00
Ta ( $\text{Mn}_{15}\text{TaSi}_{28}$ )	0.00
Cr ( $\text{Mn}_{15}\text{CrSi}_{28}$ )	0.97
Mo ( $\text{Mn}_{15}\text{MoSi}_{28}$ )	0.98
W ( $\text{Mn}_{15}\text{WSi}_{28}$ )	0.93
Re ( $\text{Mn}_{15}\text{ReSi}_{28}$ )	0.00
B ( $\text{Mn}_{16}\text{Si}_{27}\text{B}$ )	0.91
Al ( $\text{Mn}_{16}\text{Si}_{27}\text{Al}$ )	0.72
Ga ( $\text{Mn}_{16}\text{Si}_{27}\text{Ga}$ )	0.66
In ( $\text{Mn}_{16}\text{Si}_{27}\text{In}$ )	0.91
C ( $\text{Mn}_{16}\text{Si}_{27}\text{C}$ )	0.00
Ge ( $\text{Mn}_{16}\text{Si}_{27}\text{Ge}$ )	0.00
Sn ( $\text{Mn}_{16}\text{Si}_{27}\text{Sn}$ )	0.00

**Table S5** The maximum ELF value for  $\text{Mn}_4\text{Si}_7$  containing extrinsic defects.

Dopant type	ELF value
None ( $\text{Mn}_{16}\text{Si}_{28}$ )	0.834
V ( $\text{Mn}_{15}\text{VSi}_{28}$ )	0.664
Nb ( $\text{Mn}_{15}\text{NbSi}_{28}$ )	0.668
Ta ( $\text{Mn}_{15}\text{TaSi}_{28}$ )	0.668
Cr ( $\text{Mn}_{15}\text{CrSi}_{28}$ )	0.670
Mo ( $\text{Mn}_{15}\text{MoSi}_{28}$ )	0.657
W ( $\text{Mn}_{15}\text{WSi}_{28}$ )	0.679
Re ( $\text{Mn}_{15}\text{ReSi}_{28}$ )	0.673
B ( $\text{Mn}_{16}\text{Si}_{27}\text{B}$ )	0.670
Al ( $\text{Mn}_{16}\text{Si}_{27}\text{Al}$ )	0.668
Ga ( $\text{Mn}_{16}\text{Si}_{27}\text{Ga}$ )	0.670
In ( $\text{Mn}_{16}\text{Si}_{27}\text{In}$ )	0.665
C ( $\text{Mn}_{16}\text{Si}_{27}\text{C}$ )	0.679
Ge ( $\text{Mn}_{16}\text{Si}_{27}\text{Ge}$ )	0.678
Sn ( $\text{Mn}_{16}\text{Si}_{27}\text{Sn}$ )	0.670