## **Supplementary Information**

## Broadband and weak-dispersion nonlinear response enhancement in

## the epsilon-near-zero region of a nano-stepped metasurface

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## **1.** The effective permittivity and electric field distribution of the metasurface deviated from the optimum structural parameters

We set  $h_2$ =140nm (112nm in the main text) and  $h_3$ =80nm (82nm in the main text). The effective permittivity of the metasurface with the unoptimized parameters is shown in Fig. S1(a). The narrowing of the ENZ waveband is accompanied by a deviation of the metasurface's effective permittivity further from zero. Figure S1(b) and (c) illustrate the electric field distribution of the metasurface at the wavelength of 1400 nm, corresponding to the ENZ point, and the wavelength-dependent variation of the electric field distribution at x=-100 nm. It is evident that a slight shift in parameters brings a slight reduction in field enhancements and nonlinear optical properties. Overall, deviations of structural parameters from the optimum values may result in a narrower ENZ band (and even absent), yet the field enhancements and nonlinear optical properties are retained within the possible ENZ band.



**Fig.S1**(a) Real (red line) and imaginary (blue line) parts of the effective permittivity of the considered metasurface with the sub-optimum parameters P = 400 nm,  $h_1 = 200$  nm,  $w_1 = 192$  nm,  $h_2 = 140$  nm,  $w_2 = 32$  nm,  $h_3 = 80$  nm,  $w_3 = 16$  nm,  $w_4 = 160$  nm. (b) Electric field distribution of the metasuface at the wavelength of 1400 nm; (c) Wavelength dependence of the electric field distribution at the x=-100 nm.