

Supporting Information

**First-principles study of valley splitting of transition-metal dichalcogenides in
 MX_2/CrI_3 (M=W, Mo; X=S, Se, Te) van der Waals heterostructures**

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Considering the mismatch between the different lattice parameters of MX_2 and CrI_3 , a twist angle between them is necessary. The red solid line in Fig. S3 indicates the supercell, and \vec{a} denotes the lattice parameter of the supercell. The lattice vector \vec{a} may be described in terms of the $\vec{a} = m\vec{a}_1 + n\vec{a}_2$, where \vec{a}_1, \vec{a}_2 are vectors of the unitcell, m, n are chiral index (as shown in Fig. S3). Here, we use θ_{layer} to represent rotated angle between the unitcell vector and supercell vector, which is defined as:

$$\theta_{\text{layer}} = \arccos \frac{a^2 + (ma_1)^2 - (na_2)^2}{2a(ma_1)}$$

The twist angle θ is defined as the difference of the rotated angle for upper and bottom layer. The parameter details corresponding to this work are listed in Table S3 as follows.

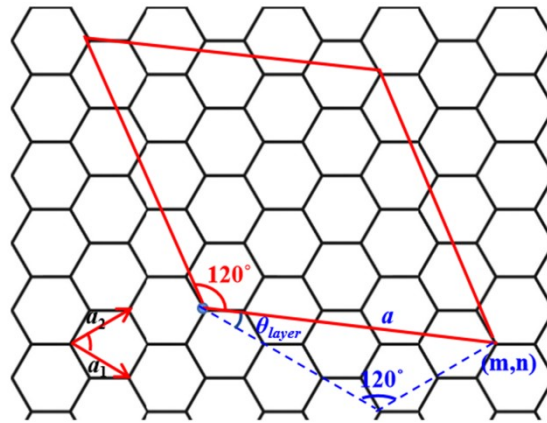


Fig. S1 Schematic diagram of supercell and twist angle.

Table S1 Details of different heterostructures. The $m(n)$, θ_{upper} (θ_{bottom}), θ , δ , l_{upper} , l_{bottom} are chiral indexes, rotated angle for upper (bottom) layer, twist angle of the two layers, lattice mismatch, and lattice of supercell for upper (bottom) layer.

	$(m,n) $ (m,n)	$\theta_{upper}(^\circ) \theta_{bottom}(^\circ)$	$\theta (^\circ)$	$\delta(\%)$	$L_{upper} l_{bottom}$
MoS ₂ /CrI ₃	(3,2) (2,0)	23.413 0	23.413	1.6	13.774 14.004
	(4,0) (1,1)	0 30	30	4.2	12.640 12.128
MoSe ₂ /CrI ₃	(2,0) (1,0)	0 0	0	6	6.58 7.002
	(3,1) (1,1)	13.898 30	16.102	2.3	11.851 12.128
MoTe ₂ /CrI ₃	(2,0) (1,0)	0 0	0	0.5	7.04 7.002
	(3,1) (1,1)	13.898 30	16.102	4.6	12.692 12.128
WS ₂ /CrI ₃	(4,0) (1,1)	0 30	30	3.9	12.6 12.128
WTe ₂ /CrI ₃	(2,0) (1,0)	0 0	0	1.6	7.12 7.002
	(3,1) (1,1)	13.898 30	16.102	5.7	12.825 12.128

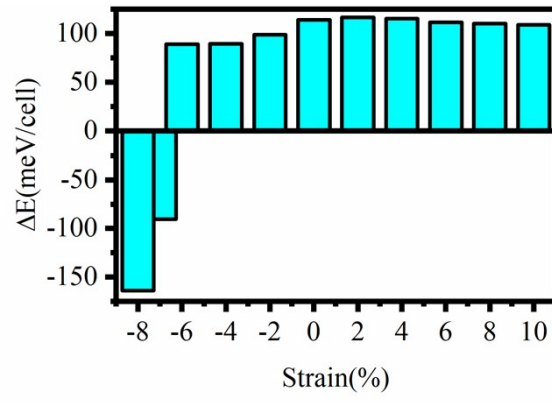


Fig. S2 Magnetic ground state of monolayer CrI_3 under different biaxial strains.

Band properties of two-dimensional (2D) transition-metal dichalcogenides (TMDs) are sensitive to strain. The band structure of free-standing MoS₂ features a direct bandgap. In the MoS₂/CrI₃ heterostructure with a twist angle of $\theta = 0^\circ$, MoS₂ experiences an 8.4% strain deformation that leads to a direct-to-indirect band transition, as illustrated in Fig. S3(b).

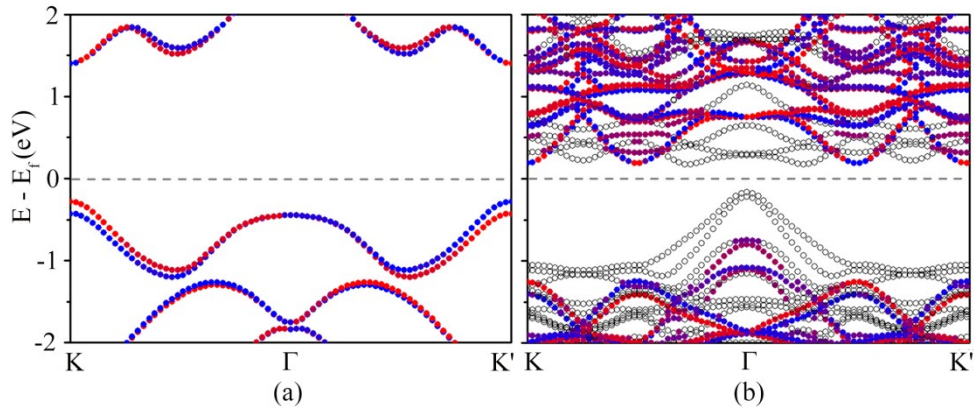


Fig. S3 Electronic structures of monolayer MoS₂ (a) and nontwisted vdW heterostructure MoS₂/CrI₃ (b) with SOC. Red and blue arrows indicate spin-up and spin-down states, respectively. Gray circles are projected bands of CrI₃ layer.

Table S2. Average magnetic moments of Cr ions in 2D vdW heterostructures MX₂/CrI₃, corresponding to Table 1 in the manuscript.

	a (Å)	θ (°)	Cr_{tot} (μ_B)	Cr_s (μ_B)	Cr_p (μ_B)	Cr_d (μ_B)
MoS ₂ /CrI ₃	13.770	23.413	3.321	0.042	0.048	3.231
	12.570	30	3.302	0.04	0.036	3.225
MoSe ₂ /CrI ₃	6.660	0	3.29	0.041	0.048	3.20
	11.868	16.102	3.320	0.042	0.048	3.23
MoTe ₂ /CrI ₃	7.0	0	3.360	0.043	0.048	3.27
	12.527	16.102	3.21	0.037	0.033	3.14
WS ₂ /CrI ₃	12.556	30	3.395	0.042	0.048	3.30
WTe ₂ /CrI ₃	7.0	0	3.364	0.043	0.048	3.273

Table S3. Average magnetic moments of Cr ions in 2D vdW heterostructures MX₂/CrI₃, corresponding to Table 2 in the manuscript.

	a (Å)	θ (°)	Cr_{tot} (μ_B)	Cr_s (μ_B)	Cr_p (μ_B)	Cr_d (μ_B)
MoS ₂ /CrI ₃	13.774	23.413	3.322	0.042	0.048	3.232
	12.640	30	3.312	0.04	0.036	3.24
MoSe ₂ /CrI ₃	6.574	0	3.282	0.041	0.047	3.193
	11.851	16.102	3.316	0.042	0.048	3.226
MoTe ₂ /CrI ₃	7.004	0	3.175	0.036	0.033	3.105
	12.692	16.102	3.231	0.037	0.033	3.16

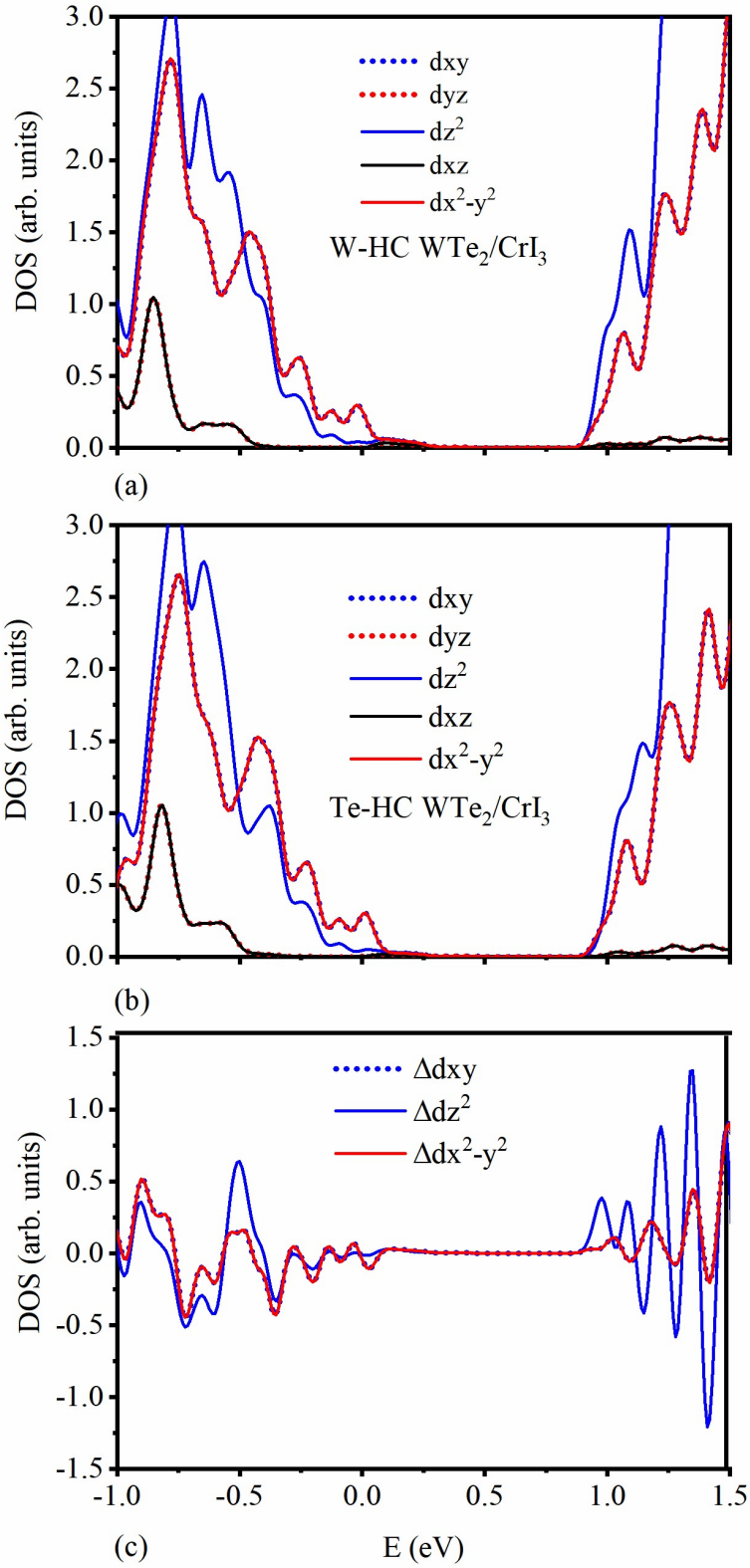


Fig. S4 Projected Density-of-States of W atoms in 2D heterostructures (a) W-HC WTe_2/CrI_3 , (b) Te-HC WTe_2/CrI_3 and (c) the differences in DOS between (a) and (b) for orbitals d_{xy} , d_{z^2} and $d_{x^2-y^2}$.