## Quantum spins Hall states in $MX_2$ (M = Ru, Os; X = As, Sb)

## monolayers

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**Table S1** Lattice parameters of the predicted RuAs<sub>2</sub>, RuSb<sub>2</sub>, OsAs<sub>2</sub>, and OsSb<sub>2</sub> monolayers.

	Lattice parameters (Å)			
Phase	1T'	1T	2Н	
RuAs <sub>2</sub>	a = 3.062, b = 6.360	a = b = 3.109	a = b = 2.978	
	$\alpha = 90^{\circ}$	$\alpha = 60^{\circ}$	$\alpha = 60^{\circ}$	
RuSb <sub>2</sub>	a = 3.273, b = 6.648	a = b = 3.325	a = b = 3.222	
	$\alpha = 90^{\circ}$	$\alpha = 60^{\circ}$	$\alpha = 60^{\circ}$	
OsAs <sub>2</sub>	a = 3.093, b = 6.358	a = b = 3.148	a = b = 3.009	
	$\alpha = 90^{\circ}$	$\alpha = 60^{\circ}$	$\alpha = 60^{\circ}$	
OsSb <sub>2</sub>	<i>a</i> = 3.285, <i>b</i> = 6.635	a = b = 3.352	a = b = 3.217	
	$\alpha = 90^{\circ}$	$\alpha = 60^{\circ}$	$\alpha = 60^{\circ}$	

**Table S2** Calculated cohesive energies for different phases of  $MX_2$  (M = Ru, Os; X = As, Sb) monolayers.

	RuAs <sub>2</sub>	RuSb <sub>2</sub>	OsAs <sub>2</sub>	OsSb <sub>2</sub>
1T′	-6.17	-5.70	-6.69	-6.16
2H	-6.16	-5.66	-6.67	-6.09
1T	-6.04	-5.57	-6.53	-6.00

**Table S3** Calculated energy oscillation intervals for different  $MX_2$  (M = Ru, Os; X =

As,	Sb)	monolayers	after	heating	at 300K	and	500K.
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Energy interval (eV)	RuAs <sub>2</sub>	RuSb <sub>2</sub>	OsAs <sub>2</sub>	OsSb <sub>2</sub>
300K	0.47	0.55	0.54	0.49
500K	0.91	0.70	0.87	0.75



Figure S1 Phonon dispersion of (a)  $RuSb_2$ , (b)  $OsAs_2$ , and (c)  $OsSb_2$  monolayers,

indicating that the structures are dynamically stable.



**Figure S2** Phonon dispersion of (a) RuAs<sub>2</sub>, (b) RuSb<sub>2</sub>, (c) OsAs<sub>2</sub>, and (d) OsSb<sub>2</sub> monolayers after heating at 300K for 5 ps. Phonon dispersion of (e) RuAs<sub>2</sub>, (f) RuSb<sub>2</sub>, (g) OsAs<sub>2</sub>, and (h) OsSb<sub>2</sub> monolayers after heating at 500K for 5 ps.



**Figure S3** Band structures of (a) RuSb<sub>2</sub>, (b) OsAs<sub>2</sub>, and (c) OsSb<sub>2</sub> monolayers without SOC.



**Figure S4** Band structures of (a)  $RuSb_2$ , (b)  $OsAs_2$ , and (c)  $OsSb_2$  monolayers without SOC. Band structures of (d)  $RuSb_2$ , (e)  $OsAs_2$ , and (f)  $OsSb_2$  monolayers with SOC. All results are calculated using the hybrid functional (HSE06) method.



**Figure S5** Evolution of the Wannier charge center (WCC) for (a)  $RuSb_2$ , (b)  $OsAs_2$ , and (c)  $OsSb_2$  monolayers along *k*. Topological edge states of the semi-infinite (d)  $RuSb_2$ , (e)  $OsAs_2$ , and (f)  $OsSb_2$  monolayers with SOC. All results are calculated using the hybrid functional (HSE06) method.



**Figure S6** Calculated band structures of  $\operatorname{RuSb}_2$  monolayers in the strain range from -5% to -1% in the (a) *x* direction and (c) *y* directions, and from 1% to 5% in the (b) *x* direction and (d) *y* directions.



**Figure S7** Calculated band structures of  $OsSb_2$  monolayers in the strain range from -5% to -1% in the (a) *x* direction and (c) *y* directions, and from 1% to 5% in the (b) *x* direction and (d) *y* directions.