## Singlet Fission in Carotenoid Dimers -The Role of the Exchange and Dipolar Interactions Supplementary Information

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## 1. Matrix Representation of the Reduced Two-Triplet Hamiltonian, eqn. (29)

Here we give the matrix representation, in the spin-coupled basis  $|S, M_S\rangle$ , of the exchange interactions (introduced in the reduced two-triplet Hamiltonian (eqn. (29) in the main paper)), and the dipolar and Zeeman interactions. We take the axis of spin quantisation to be parallel to the principal axis **Z**. We use the notation  $B_i \equiv B_x + iB_y$  and  $B_i^* \equiv B_x - iB_y$ . The order of the basis functions is

 $\{|0,0\rangle, |2,0\rangle, |2,-2\rangle, |2,+2\rangle, |2,-1\rangle, |2,+1\rangle, |1,0\rangle, |1,-1\rangle, |1,+1\rangle\},\$ 

where the basis functions are defined by eqn. (2) of the main paper (taking  $i \equiv A$  and  $j \equiv B$ ).

$$\mathbf{H}_{dipolar} = \begin{bmatrix} 0 & \frac{\sqrt{8}}{3}D & \frac{2}{\sqrt{3}}E & \frac{2}{\sqrt{3}}E & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{8}}{3}D & -\frac{2}{3}D & \frac{2}{\sqrt{6}}E & \frac{2}{\sqrt{6}}E & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}}E & \frac{2}{\sqrt{6}}E & \frac{2}{3}D & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}}E & \frac{2}{\sqrt{6}}E & 0 & \frac{2}{3}D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{3}D & E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E & -\frac{1}{3}D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3}D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3}D & -E \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -E & -\frac{1}{3}D \end{bmatrix}$$
(1)

		0	0	0	0	0	0	0	0	0	
		0	0	0	0	$\frac{3}{\sqrt{6}}B_i^*$	$\frac{3}{\sqrt{6}}B_i$	0	0	0	
		0	0	$-2B_z$	0	$B_i$	0	0	0	0	
		0	0	0	$2B_z$	0	$B_i^*$	0	0	0	
$rac{\mathbf{H}_{ ext{Zeeman}}}{\mu_B g} =$		0	$\frac{3}{\sqrt{6}}B_i$	$B_i^*$	0	$-B_z$	0	0	0	0	(2)
		0	$\frac{3}{\sqrt{6}}B_i^*$	0	$B_i$	0	$B_z$	0	0	0	
		0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}B_i^*$	$\frac{1}{\sqrt{2}}B_i$	
		0	0	0	0	0	0	$\frac{1}{\sqrt{2}}B_i$	$-B_z$	0	
		0	0	0	0	0	0	$\frac{1}{\sqrt{2}}B_i^*$	0	$B_z$	
	$-2J_1 - 4J_2$		0	0	0	0	0	0	0	0 -	]
$_{\rm Hexchange} =$	0		$J_1 - J_2$	0	0	0	0	0	0	0	
	0		0	$J_1 - J_2$	0	0	0	0	0	0	
	0		0	0	$J_1 - J_2$	0	0	0	0	0	
	0		0	0	0	$J_1 - J_2$	0	0	0	0	(3)
	0		0	0	0	0	$J_1 - J_2$	0	0	0	
	0		0	0	0	0	0	$-J_1 - J_2$	0	0	
	0		0	0	0	0	0	0	$-J_1 - J_2$	0	
	0		0	0	0	0	0	0	0	$-J_1 - J_2$	J

## 2. Derivation of the Lindblad Rate Constants

Below we present the derived relative rates of population transfer between the coupled-spin states  $|S, M_S\rangle$ . We use the notation  $\gamma_1 \equiv \frac{1}{T_1}$  and  $\gamma_2 \equiv \frac{1}{T_2}$ . The relative rates are the values that the multiplicative factor  $r_p$  takes in the main text (see Section 2.3 and eqn. (22)).



Figure 1: The derived relative rates of population transfer (i.e.,  $r_p$ ) between the spin-coupled basis states as a result of longitudinal spin-dephasing.



Figure 2: The derived relative rates of population transfer (i.e.,  $r_p$ ) between the spin-coupled basis states as a result of transverse spin-dephasing.