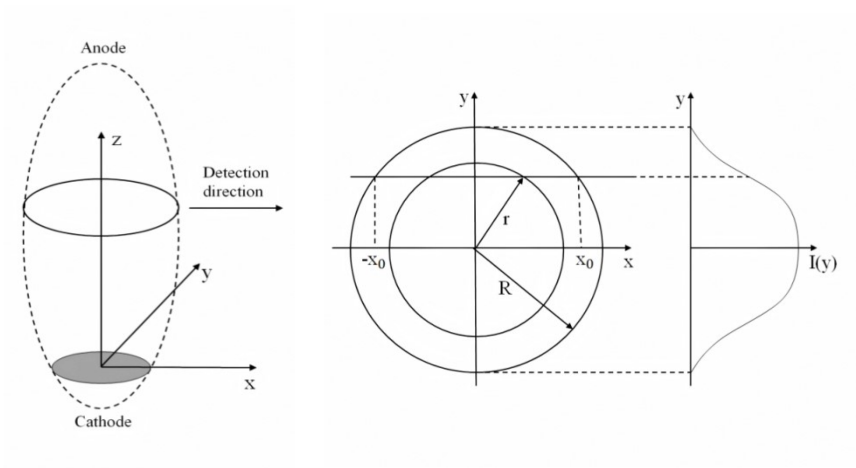
Supplementary materials

Fig. 2 is a geometrical interpretation of the Abel transform in a cylindrically symmetric plasma. As shown in Fig. 2(a), assuming that the plasma radiation source is a cylindrical body with cylindrical symmetry about the z-axis and is optically thin. In Fig. 2(b), the detector moves in the y-direction, allowing for the acquisition of the transverse distribution of plasma spectral intensity $I(y)$.



(a) (b)

Fig.2. Geometrical interpretation of the Abel transform in a cylindrically symmetric plasma.

(a)Cylindrical symmetric plasma, (b)Transverse intensity.

The relationship between measured intensity $I(y)$ and radial emissivity $E(r)$ is described by Abel inversion:

$$I(y) = \int_{-x_0}^{x_0} E(r)dx = 2\int_{y}^{1} \frac{E(r)r}{\sqrt{r^2 - y^2}}dr \tag{1}$$

where, y is the offset of the intensity profile from the central axis of the plasma, r is the radial distance from the source center ($r = \sqrt{x^2 + y^2}$), and x_0 is the x-coordinate of the plasma edge at each value of y as illustrated in the Fig. 2. The radial emissivity $E(r)$ is obtained through Abel inversion as follows:

$$E(r) = -\frac{1}{\pi}\int_r^R \frac{dI(y)}{dy}\frac{dy}{\sqrt{y^2 - r^2}} \tag{2}$$

All Abel inversion algorithms face two main issues. The first issue is that the experimentally measured $I(y)$ often contains random noise, making it difficult to obtain the first derivative $I'(y)$. The other issue is that the Abel inversion formula involves a singularity at the lower limit of the integral and requires the differentiation of $I(y)$. The cubic spline method was used to solve the problems. The cubic spline method approximates the data points (y_i, I_i) and (y_{i+1}, I_{i+1}) with a cubic polynomial, as follows:

$$I_i(y) = a_{i0} + a_{i1}y + a_{i2}y^2 + a_{i3}y^3 \tag{3}$$

On the subinterval [y_i, y_{i+1}], $I(y) = I_i(y)$ is a cubic polynomial, so $I''(y)$ is a linear function of x. Given $I''(y_{i+1}) = M_{i+1}$ and $I''(y_i) = M_i$, linear interpolation provides that:

$$I''(y) = M_i\frac{y_{i+1} - y}{y_{i+1} - y_i} + M_{i+1}\frac{y - y_i}{y_{i+1} - y_i} \tag{4}$$

The above expression is integrated twice, using the boundary values obtained I_{i+1} and I_i from the interpolation.

$$I(y) = \frac{(y_{i+1} - y)^3}{6(y_{i+1} - y_i)}M_i + \frac{(y - y_i)^3}{6(y_{i+1} - y_i)}M_{i+1} + \left(I_i - \frac{M_i}{6}(y_{i+1} - y_i)^2\right)\frac{y_{i+1} - y}{y_{i+1} - y_i}$$
$$+ (I_{i+1} - \frac{M_{i+1}}{6}(y_{i+1} - y_i)^2)\frac{y - y_i}{y_{i+1} - y_i} \tag{5}$$

Let $h_i = y_{i+1} - y_i$, the expression can be written as follows:

$$a_{i0} = \frac{M_i y_{i+1}^3 - M_{i+1}y_i^3}{6h_i} + \frac{(I_i - \frac{M_i h_i^2}{6})y_{i+1}}{h_i} - \frac{(I_{i+1} - \frac{M_{i+1}h_i^2}{6})y_i}{h_i} \tag{6}$$

$$a_{i1} = \frac{M_{i+1}y_i^2 - M_i y_{i+1}^2}{2h_i} + \frac{(I_i - \frac{M_i h_i^2}{6})}{h_i} - \frac{(I_{i+1} - \frac{M_{i+1}h_i^2}{6})}{h_i} \tag{7}$$

$$a_{i2} = \frac{M_i y_{i+1} - M_{i+1} y_i}{2h_i} \tag{8}$$

$$a_{i3} = \frac{M_{i+1} - M_i}{6h_i} \tag{9}$$

To solve for the coefficients a_{i0}, a_{i1}, a_{i2}, and a_{i3} of the cubic polynomial, it is first necessary to determine the n+1 unknown values: M_1,…, M_{i+1}. The following conditions must be satisfied to ensure continuity at the nodes:

$$I_{i-0}^{'} = I_{i+0}^{'} \tag{10}$$

There are n-1 nodes between the measurement points, which gives rise to n-1 linear algebraic equations based on the above conditions. Assuming that the plasma has an axisymmetric shape, the first derivative of the intensity at the center position is assumed to be zero. Additionally, the emissivity at the plasma boundary, where r = R, is also assumed to be zero, which implies that the corresponding first derivative at this boundary is also zero.

$$I^{'}(0) = I^{'}(R) = 0 \tag{11}$$

This results in two additional equations. Thus, the n+1 linear equations allow for the determination of n+1 second-order derivatives *M*. By substituting these derivatives, the coefficients of the cubic polynomials can be calculated. Substituting these coefficients will then yield the radial emissivity distribution $E(r)$ of the plasma.