Supplementary Materials

An enhanced heat transfer method based on the electrocapillary effect of gallium-based liquid metal

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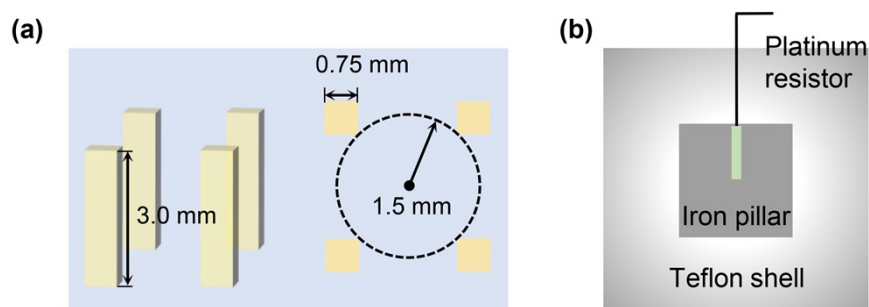
S1 Experimental system

Fig. S1 (a) The structure and size of pillars; (b) Schematic diagram of the position of iron pillars, Teflon sleeves, and platinum resistors.

S2 Experimental data processing

Fig. S2(a) shows the experimental images taken by a high-speed camera. The experimental images are superimposed using Matlab to obtain the trajectories of the graphite particles as shown in Fig.
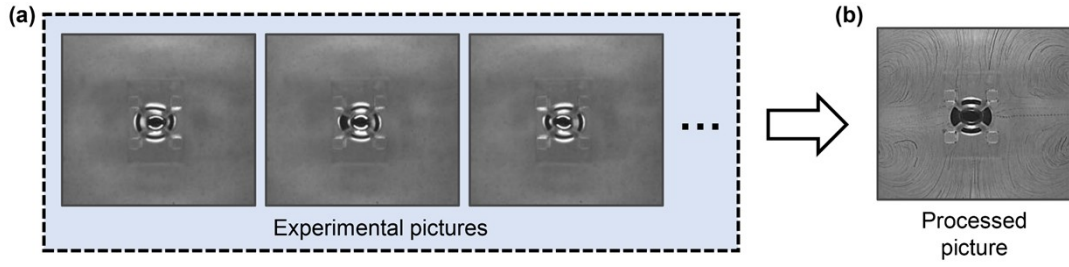
S2(b).

Fig. S2 (a) Experimental image; (b) Processed image.

S3 Heat flow calculation for natural convection process

The heat transfer coefficient h between the solution and the metal surface and the air during natural convection is approximately 5~25 W/(m²·K), Take the maximum value of 25 W/(m²·K) when estimating. From Newton's cooling equation, we know that the heat flow Q_{s-a} between the solution and the air is

$$Q_{s-a} = A_{s-a} h \Delta T_{s-a} = 0.0004 \times 25 \times 7 = 0.07 \text{ W} \qquad \backslash* \text{ MERGEFORMAT (1)}$$

where A_{s-a} is the contact area between the solution and the air, and ΔT_{s-a} is the temperature difference between the solution and the air, which is approximately 7°C.

The heat flow Q_{e-a} between the metal surface and the air is

$$Q_{e-a} = A_{e-a-1} h \Delta T_{e-a-1} + A_{e2} h \Delta T_{e-a-2} = A_{e-a-1} h \left(\Delta T_{e-a-1} + \Delta T_{e-a-2} \right) \backslash* \text{ MERGEFORMAT (2)}$$
$$= 0.000365 \times 25 \times \left(-10 + 20 \right) = 0.09 \text{ W}$$

where A_{e-a-1} and A_{e-a-2} are the contact areas between the hot and cold end electrode plates and the air, respectively; ΔT_{e-a-1} and ΔT_{e-a-2} are the temperature differences between the hot and cold end electrode plates and the air, respectively, which are approximately -10 and 20°C. The heat flow transfers to the cold end through the iron pillar:

$$Q = -\lambda_{Fe} A_p \frac{\Delta T_{3-1}}{\Delta x_{3-1}} = -80 \times 0.0004 \times \frac{1.45}{0.02} = 2.3 \text{ W} \qquad \backslash* \text{ MERGEFORMAT (3)}$$

where λ_{Fe} is the thermal conductivity of iron, A_p is the cross-sectional area of the iron pillar, ΔT_{3-1} is the temperature difference between position No. 3 and No. 1 in the pillar, and Δx_{3-1} is the distance between two positions No. 3 and No. 1.

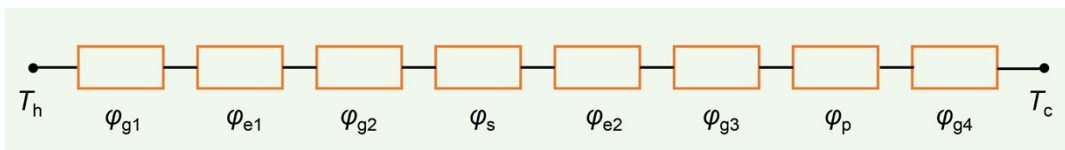## S4 Thermal resistance network diagram of the experimental system

S5 Calculation of the velocity of the solution on the droplet surface

The velocity of the droplet surface is deduced by referring to our previous work.[1] First, the surface tension gradient of the droplet is determined by the potential distribution. For analysis, the solution region is divided into three parts as shown in Fig. S4(a). $L_1\sim L_3$ are the lengths of each region, and $L_1=L_3$, $L_2=2r$, and the corresponding resistances are $R_1\sim R_3$. e is the width of the solution region, and r is the radius of the droplet. Fig. S4(b) is the equivalent circuit of the solution region and the electrode plates,[2, 3] where C_{e1} and C_{e2} are the capacitances of the electric double layer (EDL) between the electrode plates and the solution, R_{e1} and R_{e2} are the transfer resistance of the charges of the EDL between the electrode plates and the solution. V_E is the voltage applied to the electrode plates.
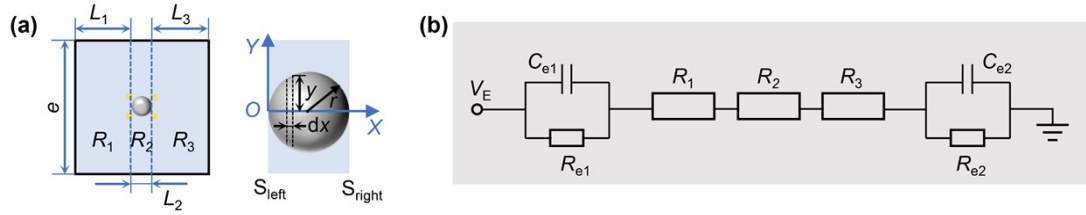

Fig. S4 (a) The resistance division of the solution region; (b) The equivalent circuit diagram of the solution region and the electrode plates.

The resistance of each solution region is proportional to the length and inversely proportional to the cross-sectional area of the region. When the solution depth is d and the conductivity is σ, the resistances are:

$$R_1 = R_3 = \frac{L_1}{\sigma ed} = \frac{e-2r}{2\sigma ed} \qquad \text{\\* MERGEFORMAT (S4)}$$

For the resistance of the droplet region, R_2, in Fig.S4, the resistance of the solution between the EGaIn droplet and the channel wall is mainly paid attention to, which only depends on the properties of the solution. R_2 is determined by the integration method:

$$R_2 = \frac{1}{\sigma}\int_0^{2r}\frac{dx}{A_s} = \frac{1}{\sigma}\int_0^{2r}\frac{dx}{ed - \pi\left[r^2 - (x-r)^2\right]}$$

$$= \frac{2}{\pi\sigma}\sqrt{\frac{\pi}{ed-\pi r^2}}\arctan\left(r\sqrt{\frac{\pi}{ed-\pi r^2}}\right) \qquad \text{\\* MERGEFORMAT (S5)}$$

For ease of the expression, denote $k = \sqrt{\pi/(ed-\pi r^2)}$, and eqn (S5) is simplified:

$$R_2 = \frac{2k}{\pi\sigma}\arctan(kr) \qquad \text{\textbackslash* MERGEFORMAT (S6)}$$

The resistance of the solution between the tangent planes on the left of the droplet S_{left} and the x-section (i.e., $0{\sim}x$) is:

$$
\begin{aligned}
R_x &= \frac{1}{\sigma}\int_0^x \frac{dx}{A_s} = \frac{1}{\sigma}\int_0^x \frac{dx}{ed - \pi\left[r^2 - (x-r)^2\right]} \qquad \text{\textbackslash* MERGEFORMAT (S7)}\\
&= \frac{k}{\pi\sigma}\left\{\arctan\left[k(x-r)\right] + \arctan(kr)\right\}
\end{aligned}
$$

where A_s is the cross-sectional area of the solution. The surface tension γ of the droplet is related to the capacitance c per unit area of the EDL and the potential difference $V_{\text{t-}x}$ across the EDL:

$$\gamma = \gamma_0 - \frac{1}{2}cV_{\text{t-}x}^{\ 2} \qquad \text{\textbackslash* MERGEFORMAT (S8)}$$

where γ_0 is the surface tension of the droplet when $V_{\text{t-}x}=0$. The potential difference $V_{\text{t-}x}$ of the EDL of the droplet is the sum of the potential difference V_0 at each point in the original electric double layer of the droplet and the potential difference V_x of the induced double layer at any position x on the droplet surface:

$$V_{\text{t-}x} = V_0 + V_x \qquad \text{\textbackslash* MERGEFORMAT (S9)}$$

where

$$V_0 = \frac{q_0}{c} \qquad \text{\textbackslash* MERGEFORMAT (S10)}$$

$$V_x = \frac{k\arctan\left[k(r-x)\right](V_{\text{E}} - 2V_{\text{RC}})}{\pi\sigma(R_1 + R_2 + R_3)} \qquad \text{\textbackslash* MERGEFORMAT (S11)}$$

q_0 is the charge density in the original electric double layer of the droplet, and V_{RC} is the potential difference across the RC circuit.

The shear stress at x on the surface of the droplet:

$$\tau_x = \frac{\partial\gamma}{\partial x} \qquad \text{\textbackslash* MERGEFORMAT (S12)}$$

Combining eqs (S8) - (S12), eqn (S12) can be written as:

$$\tau_x = \left[\frac{k\arctan\left[k(r-x)\right](V_{\text{E}} - 2V_{\text{RC}})}{\pi\sigma(2R_1 + R_2)} + \frac{q_0}{c}\right]\frac{c(V_{\text{E}} - 2V_{\text{RC}})}{\sigma(2R_1 + R_2)\left[ed + \pi x(x - 2r)\right]} \text{MERGEFORMAT}$$

(S13)

Therefore, the flow velocity of the solution with a dynamic viscosity of μ at any x on the droplet surface is:

$$u_x = \frac{\tau_x r}{\mu} = \left[\frac{k\arctan\left[k(r-x)\right](V_{\text{E}} - 2V_{\text{RC}})}{\pi\sigma(2R_1 + R_2)} + \frac{q_0}{c}\right]\frac{cr(V_{\text{E}} - 2V_{\text{RC}})}{\mu\sigma(2R_1 + R_2)\left[ed + \pi x(x - 2r)\right]} \text{MERGEFORMAT}$$

(S14)

When the number of droplets is n, R_2 can be expressed:

$$R_2 = \frac{1}{\sigma}\int_0^{2r}\frac{\mathrm{d}x}{A_s} = \frac{1}{\sigma}\int_0^{2r}\frac{\mathrm{d}x}{ed - n\pi\left[r^2 - (x-r)^2\right]}$$
$$= \frac{2}{n\pi\sigma}\sqrt{\frac{n\pi}{ed - n\pi r^2}}\arctan\left(r\sqrt{\frac{n\pi}{ed - n\pi r^2}}\right) \qquad \text{* MERGEFORMAT (S15)}$$

Denote $k' = \sqrt{n\pi/(ed - n\pi r^2)}$, and eqn (S15) is:

$$R_2 = \frac{2k'}{n\pi\sigma}\arctan\left(k'r\right) \qquad \text{* MERGEFORMAT (S16)}$$

R_x is:

$$R_x = \frac{1}{\sigma}\int_0^x\frac{\mathrm{d}x}{A_s} = \frac{1}{\sigma}\int_0^x\frac{\mathrm{d}x}{ed - n\pi\left[r^2 - (x-r)^2\right]}$$
$$= \frac{k'}{n\pi\sigma}\left\{\arctan\left[k'(x-r)\right] + \arctan\left(k'r\right)\right\} \qquad \text{* MERGEFORMAT (S17)}$$

Therefore, the velocity of the solution at any position x on the surface of the droplet is:

$$u_x = \frac{\tau_x r}{\mu} = \left[\frac{k'\arctan\left[k'(r-x)\right]\left(V_E - 2V_{RC}\right)}{n\pi\sigma\left(2R_1 + R_2\right)} + \frac{q_0}{c}\right]\frac{cr\left(V_E - 2V_{RC}\right)}{\mu\sigma\left(2R_1 + R_2\right)\left[ed + n\pi x(x-2r)\right]} \qquad \text{* MERGEFORMAT (S}$$

S6 Distribution of the temperature of the solution at the final state under the action of two droplets
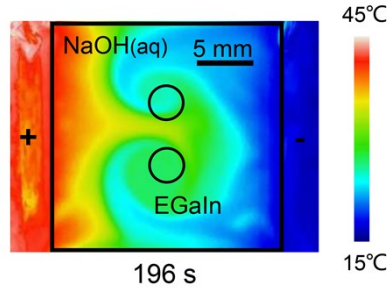

Fig. S5 The temperature distribution of the solution at the final state under the action of two droplets; V_{p-p}=10 V, V_{DC}=5 V, f=100 Hz.

References

1. L. Dai, X.M. Wu, H. Hou, Z. Hu, Y. Lin and Z. Yuan, *Lab Chip*, 2024, **24**, 1977-1986.
2. S. Y. Tang, V. Sivan, K. Khoshmanesh, A. P. O'Mullane, X. Tang, B. Gol, N. Eshtiaghi, F. Lieder, P. Petersen, A. Mitchell and K. Kalantar-zadeh, *Nanoscale*, 2013, **5**, 5949-5957.

3. S.-Y. Tang, K. Khoshmanesh, V. Sivan, P. Petersen, A. P. O'Mullane, D. Abbott, A. Mitchell and K. Kalantar-Zadeh, *Proc. Natl. Acad. Sci. U. S. A.*, 2014, **111**, 3304-3309.