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Supplementary Information for *Pervaporation-driven electrokinetic energy harvesting using poly(dimethylsiloxane) microfluidic chips*

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1 Mask of the channel network

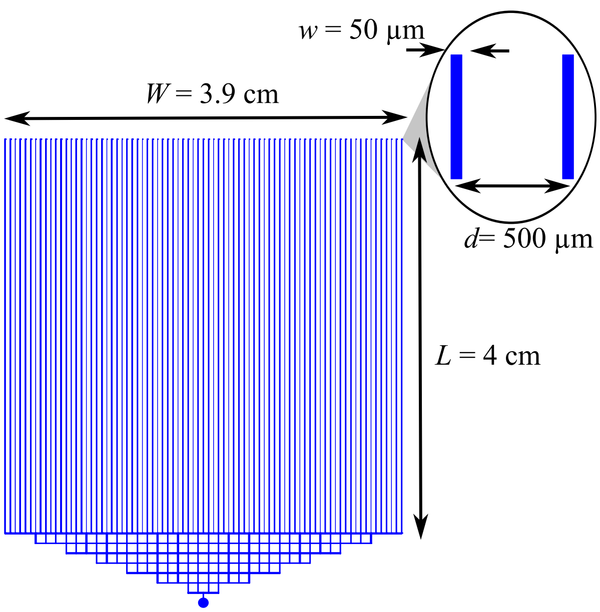

Fig. S1 Mask used to make the leaf channel network. $N = 79$ dead-end microchannels of width $w = 50$ μm and length $L = 4$ cm are connected to a single inlet. The centre-to-centre distance between adjacent channels is $d = 500$ μm resulting in a total leaf width of $W = 3.9$ cm.

2 Approximations for the pervaporation-driven flow rate of a single channel

The relation:

$$F \simeq \frac{\pi}{\log(16H/(\pi w))}, \tag{S1}$$

derived in another context[1] and only valid for $H \gg w \gg h$, provides nevertheless a rough approximation of the analytical solution given by Dollet et al.[2] for $H \gg 1$ mm, see Fig. 4. Eqn S1 shows the weak logarithmic dependence of the pervaporation-driven flow rate with the transverse dimensions of the channel as early recognised for thick chips[3,4]. For thinner chips, Dollet *et al.* provided the following approximation[2]:

$$F \simeq \frac{w}{\delta} + \frac{2}{\pi}\left[\ln\frac{(H+\delta)h}{\delta^2} + \frac{H}{\delta}\ln\frac{H+\delta}{h}\right], \tag{S2}$$

with $\delta = H - h$, valid for $\delta \leq w$ and when h is not too small compared to w. As shown in Fig. 4, eqn S2 correctly approximates the analytical solution in our configuration even for $H \leq 200$ μm.

3 Numerical estimate of the pervaporation rate

To estimate the pervaporation rate Q for the channel network shown in Fig. S1, we performed numerical resolutions to calculate the concentration field C (kg/m^3) of water within the PDMS leaf and the corresponding water mass flux ∇C, see Refs.[5,6] for similar calculations. Fig. S2(a) shows the cross-section of the leaf which comprises N identical channels of rectangular cross-section $h \times w$. The problem is nearly invariant along the channels because $L \gg H$, thus justifying a 2D description. Because of the symmetries, we only solve the steady diffusion equation for C inside the dotted rectangle shown in Fig. S2(a). We then assume that the water concentration C in PDMS follows Henry's law and that the water diffusion coefficient D_w (m^2 s^{-1}) in the matrix is constant (reasonable approximations according to Refs.[2,7]). With these assumptions, one can solve the dimensionless 2D diffusion equation $\Delta\tilde{c} = 0$, with $\tilde{c} = (C/C_{\text{sat}} - \text{RH})/(1-\text{RH})$, C_{sat} (kg m^{-3}) being the concentration of water at saturation in PDMS. The parameter $\tilde{q}$ (m^2 s^{-1}) given in the main text is then $\tilde{q} = D_w C_{\text{sat}}/\rho_w$, with ρ_w (kg m^{-3}) the water density[8]. With this definition, boundary conditions are $\tilde{c} = 1$ at the channel walls and $\tilde{c} = 0$ at the air/PDMS interface, and no-flux on the other boundaries for reasons of symmetry and because glass is impermeable to water.

Fig. S2(b) shows the numerical resolution of the concentration field $\tilde{c}$ for the case studied in the present work: $h = 30$, $w = 50$, $H = 200$, and $d = 500$ μm (Matlab, pde toolbox). The water flux normal to the air/PDMS interface is numerically estimated from such concentration maps, and is used to finally estimate the overall pervaporation rate Q induced by the leaf by summing the N channels of length L. Fig. S3 shows the pervaporation-driven flow rate Q calculated using these numerical resolutions for a PDMS leaf of thickness $H = 200$ μm and width $W = 3.9$ cm as a function of the number of channels N it contains, and thus

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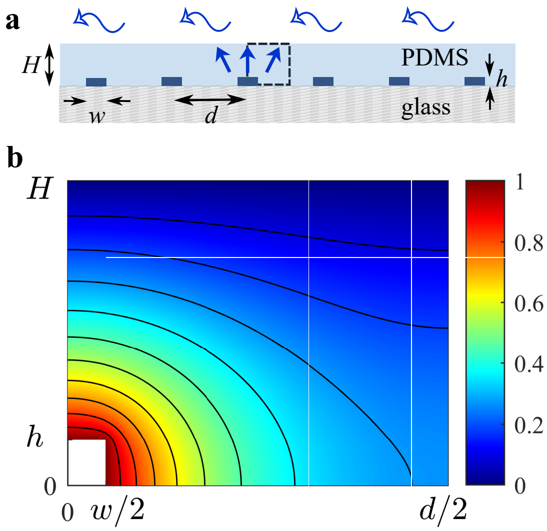

Fig. S2 (a) Cross-section view of the PDMS leaf. The dotted rectangle shows the calculation domain of the diffusion equation governing the concentration of water in PDMS. Concentration field $\tilde{c}$ estimated numerically for $h = 30$, $w = 50$, $H = 200$, and $d = 500$ μm (the thin dark lines are isoconcentration lines).

for varying center-to-center distance d between adjacent channels ($h = 30$ and $w = 50$ μm). Q is normalized by the limiting pervaporation-driven flow rate Q_{lim} corresponding to the high channel density regime $1/d \gg 1/H$ (strong coupling between the adjacent channels, see eqn 2). For $N \ll 100$, Q increases linearly with N, and saturates at Q_{lim} for $N \gg 100$. The experimental case studied in the present work ($N = 79$) is in between these two asymptotic regimes, and the numerical resolution gives in that case $Q = \alpha Q_{\text{lim}}$, with $\alpha \simeq 0.5$. We refer the reader to Ref.[5] for a full description and in-depth discussion of this problem.

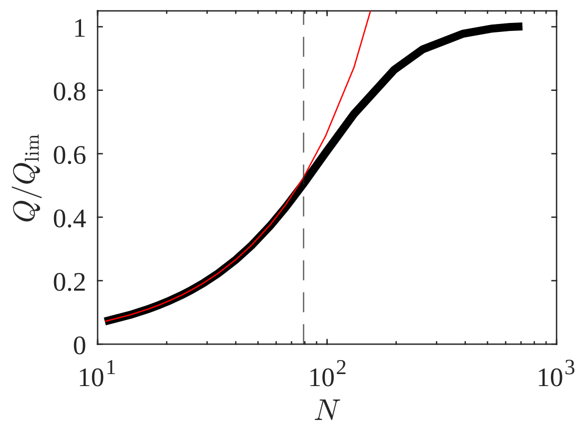

Fig. S3 Pervaporation-driven flow rate Q normalized by Q_{lim} (eqn 2) for a leaf of thickness $H = 200$ μm and width $W = 3.9$ cm as a function of the number N of channels it contains. The red line is $Q = NQ_i$ corresponding to the low density regime ($1/d \ll 1/H$). The vertical dotted line indicates the experimental configuration studied experimentally for which $Q \simeq 0.5 Q_{\text{lim}}$.

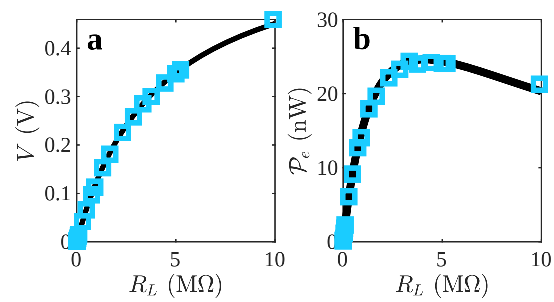

Fig. S4 (a) Streaming potential V and (b) electrical power $\mathscr{P}_e = V^2/R_L$ as a function of the load resistance R_L measured for a pressure drop $\Delta P = 1$ bar across a colloid plug of length $L_p \simeq 3$ mm in a tube with inner radius $R_t = 0.5$ mm (hydraulic resistance $R_h \simeq 0.11$ bar min μL^{-1}). The continous lines are fits by eqn 4 with $R_C = 4.1$ MΩ and $S_{\text{str}} = 155$ nA bar^{-1}.

Notes and references

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