# Quantum anomalous Hall effect in nonmagnetic bismuth monolayer with a high Chern number

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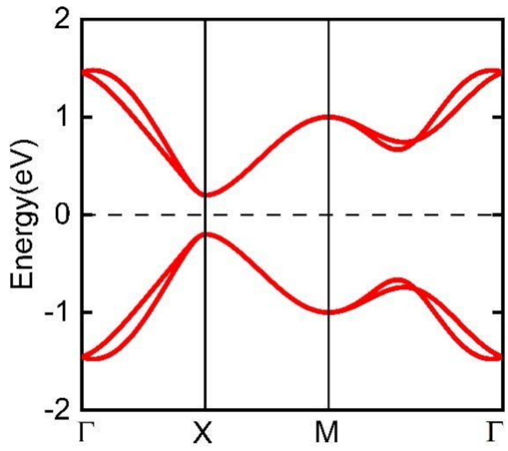


**Figure S1.** Band structure of the four-band tight-binding model for the parameters as a = 0.1 eV, t = 0.3 eV, $\lambda$ = 0.25 eV and $\lambda_{POT}$ = 0.2 eV.

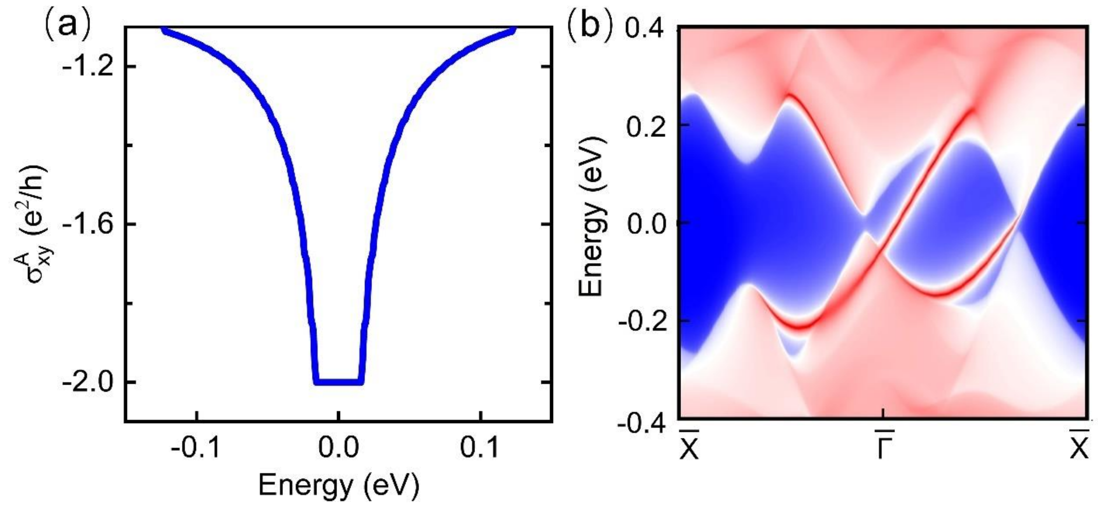


**Figure S2**. (a) Energy dependence of the anomalous Hall conductivity $\boldsymbol{\sigma_{xy}^{A}}$ and (b) the corresponding edge spectrum of TB model under the irradiation of right-handed CPL with light intensity eA/ħ = 0.15 Å$^{-1}$, indicating that the Chern number is -2.

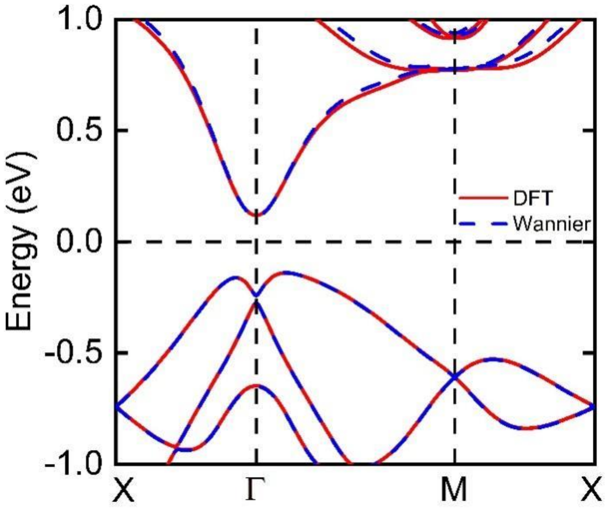


**Figure S3.** Comparison between the first-principles and Wannier fitting band structures.

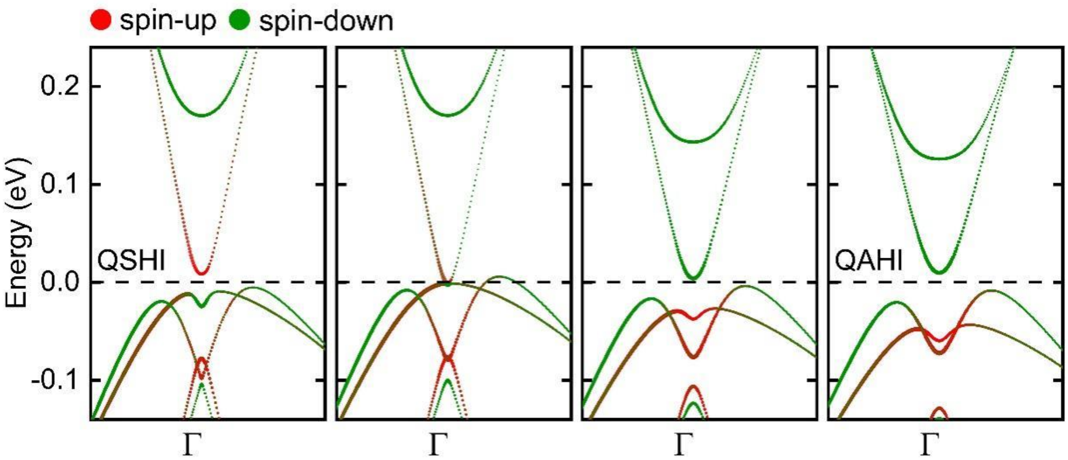


**Figure S4.** Band structure evolution for T-Bi under the irradiation of left-handed CPL with light intensity eA/$\hbar$ of 0.35, 0.37, 0.4 and 0.42 $Å^{-1}$, respectively. The red and green dots denote spin-up and spin-down component of bismuth, respectively.

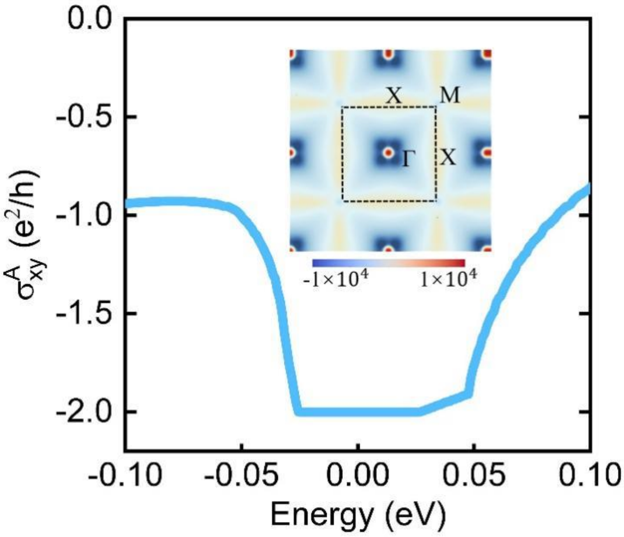


**Figure S5.** Energy dependence of the anomalous Hall conductivity $\sigma_{xy}^{A}$ under the irradiation of right-handed CPL with light intensity eA/ħ = 0.5 Å$^{-1}$, revealing a quantized value with Chern number $C$ = -2 in the gapped regime. The inset shows the reciprocal-space distribution of Berry curvatures within the gapped regime.

Note S1. Floquet Theory

We first consider it under the irradiation of an external time-dependent circularly polarized light:

$$A(\tau)=A[\eta\sin(\omega\tau)e_1+\cos(\omega\tau)e_2] \tag{S1}$$

where $\omega$ is the frequency and $A$ is the amplitude of the light, $\eta=\pm1$ represent the chirality of the circularly polarized light. The Peierls substitution $k\to k+\frac{e}{\hbar}A(\tau)$ is used to take the effect of light into account in the Hamiltonian. The time-dependent tight-binding Hamiltonian will be written as

$$H(\vec{k},\tau)=\sum_{m,n}\sum_{j}t_j^{mn}(\tau)e^{i\vec{k}\cdot\vec{R}_j}c_m^{+}(\vec{k},\tau)c_n(\vec{k},\tau) \tag{S2}$$

where $\tau$ is the time, $\vec{R}_j$ is the lattice vector and (*m, n*) is Wannier orbital index. The vector potential is coupled to the Hamiltonian through the minimal coupling of $t_j^{mn}(\tau)=t_j^{mn}e^{i\frac{e}{\hbar}\vec{A}(\tau)\cdot\vec{d}_j^{mn}}$, where $\vec{d}_j^{mn}$ is the position vector between Wannier orbital *m* in the 0-cell and Wannier orbital *n* in the *j*-cell. Taking into account the lattice and time translation invariance, the time-dependent Hamiltonian can be effectively treated with Floquet theorem by performing dual the Fourier transformation [1-2]. Then an effective static Hamiltonian in the frequency and momentum space can be obtained

$$H_F(\vec{k},\omega)=\sum_{m,n}\sum_{\alpha,\beta}[H_{\alpha-\beta}^{mn}(\vec{k})+\alpha\hbar\omega\delta_{mn}\delta_{\alpha\beta}]c_{\alpha m}^{+}(\vec{k})c_{\beta n}(\vec{k}) \tag{S3}$$

where $(\alpha,\beta)$ is the Floquet index ranging from $-\infty$ to $+\infty$, $c_{\alpha m}^{+}(c_{\beta n})$ is creation (annihilation) operator in the Floquet–Bloch picture, and the matrix element $H_{\alpha-\beta}^{mn}(\vec{k},\omega)$ is

$$H_{\alpha-\beta}^{mn}(\vec{k},\omega)=\sum_{j}e^{i\vec{k}\cdot\vec{R}_j}\left(\frac{1}{T}\int_0^T t_j^{mn}\times e^{i\frac{e}{\hbar}\vec{A}(\tau)\cdot\vec{d}_j^{mn}}e^{i(\alpha-\beta)\omega t}dt\right). \tag{S4}$$

This Floquet-Bloch Hamiltonian can be written as the block matrix form

$$H_F(\vec{k},\omega)=\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \cdots & H_0-\hbar\omega & H_{-1} & H_{-2} & \cdots \\ \cdots & H_1 & H_0 & H_{-1} & \cdots \\ \cdots & H_2 & H_1 & H_0+\hbar\omega & \cdots \\ \iddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{S5}$$

In this work, we use $q$ = 1 for the Fourier transformation, model analysis and DFT calculations. The infinite Floquet sidebands can reduce to a finite dimension by including only a few lowest order photon processes of absorbtion or emission. Here, limited to a virtual photon process, one can distinctly observe the Floquet-Bloch band structures under CPL. Each matrix element in Eq. (S5) can be obtained by averaging the time-dependent Hamiltonian.

Reference


[1] A. G´omez-Le´on and G. Platero, Phys. Rev. Lett. 110, 200403 (2013).

[2] K. F. Milfeld and R. E. Wyatt, Phys. Rev. A 27, 72 (1983).