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Tamm plasmon polariton based planar hot-electron photodetector for near-infrared region (Supplementary Information)

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1 Transfer matrix method

Transfer matrix method[1] can be used for simulation of optical properties of investigated structure. Variation in the light field upon transmission through each structure layer is determined by a second-order transfer matrix. The transfer matrix of the entire structure that relates the amplitudes of incident and transmitted waves is a product of 2x2 matrices

$$\hat{M} = \hat{T}_{0,1}\hat{T}_{1,2}...\hat{T}_{N-1,N}\hat{T}_{N,S}, \tag{1}$$

where the transfer matrix is

$$\hat{T}_{n-1,n} = \frac{1}{2}\begin{pmatrix} (1+h)\,e^{-i\alpha_n\gamma_n} & (1-h)\,e^{i\alpha_n\gamma_n} \\ (1-h)\,e^{-i\alpha_n\gamma_n} & (1+h)\,e^{i\alpha_n\gamma_n} \end{pmatrix}, \tag{2}$$

Here for TE-waves (electric field being perpendicular to the plane of incidence) $h = \sqrt{\varepsilon_n - \sin^2(\theta_0)}/\sqrt{\varepsilon_{n-1} - \sin^2(\theta_0)}, \varepsilon_n$ is the permittivity of the nth layer,
$\alpha_n = (\omega/c)\sqrt{\varepsilon(n) - \sin^2(\theta_0)}$, c is the speed of light, γ_n=z_n–z_{n-1} are the layer thicknesses (n=1,2,..., N), z_n is the coordinate of the interface between the nth layer and the $(n+1)$th layer adjacent from the right, and $\gamma_{N+1} = 0$, θ_0 is the angle of incident light. The indices 0 and S in Eq. 1 denote the media before and behind the photonic crystal (PhC). The transfer matrix for the orthogonally polarized TM-wave is obtained from Eq. (2) replacing h by new expression $h' = \varepsilon_{n-1}\sqrt{\varepsilon_n - \sin^2(\theta_0)}/\varepsilon_n\sqrt{\varepsilon_{n-1} - \sin^2(\theta_0)}$. The energy transmittances, reflectances, and absorbances are determined as

$$T(\omega) = \frac{1}{|\hat{M}_{11}|^2}, \quad R(\omega) = \frac{|\hat{M}_{21}|^2}{|\hat{M}_{11}|^2},$$

$$A(\omega) = 1 - T(\omega) - R(\omega), \tag{3}$$

where $\hat{M}_{11}, \hat{M}_{21}$ are the elements of matrix $\hat{M}$.

2 Dispersion low of the Tamm plasmon polariton

The spectral properties of the localized state, such as resonance wavelength and full width at half maximum can be determined analytically by dispersion law for Tamm plasmon polariton. Let us consider the TE-wave incident on the metal-germanium-PhC structure. A schematic representation of the structure is shown in Figure 1.

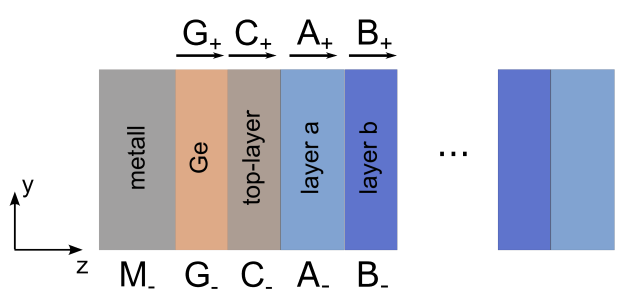

Fig. 1 Schematic representation of a one-dimensional PhC conjugated to a germanium layer and metal film.

Let's write the components of the electric and magnetic fields in each layer of the structure. The field in the metal layer can be written in the form of a wave propagating along the z axis:

$$\begin{cases} E_{my} = M_- e^{-ik_{mz}z} \\ H_{mx} = \frac{k_{mz}}{k_0} M_- e^{-ik_{mz}z}. \end{cases} \tag{4}$$

In the case of a semi-infinite metal layer, the amplitude of the

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forward wave (propagating along the positive direction z) is zero.

The field distribution in the germanium film has the form:

$$\begin{cases} E_{gy} = G_+ e^{ik_{gz}z} + G_- e^{-ik_{gz}z} \\ H_{gx} = \frac{k_{gz}}{k_0}\Big(-G_+ e^{ik_{gz}z} + G_- e^{-ik_{gz}z}\Big). \end{cases} \quad (5)$$

first PhC layer (top-layer):

$$\begin{cases} E_{cy} = C_+ e^{ik_{cz}(z-d_g)} + C_- e^{-ik_{cz}(z-d_g)} \\ H_{cx} = \frac{k_{cz}}{k_0}\Big(-C_+ e^{ik_{cz}(z-d_g)} + C_- e^{-ik_{cz}(z-d_g)}\Big). \end{cases} \quad (6)$$

The fields in the a-layer:

$$\begin{cases} E_{ay} = e^{ik\Lambda m}\Big(A_+ e^{ik_{az}(z-m\Lambda-d_c-d_g)} + \\ \qquad +A_- e^{-ik_{az}(z-m\Lambda-d_c-d_g)}\Big) \\ H_{ax} = \frac{k_{az}}{k_0} e^{ik\Lambda m}\Big(-A_+ e^{ik_{az}(z-m\Lambda-d_c-d_g)} + \\ \qquad +A_- e^{-ik_{az}(z-m\Lambda-d_c-d_g)}\Big), \end{cases} \quad (7)$$

The fields in the b layer:

$$\begin{cases} E_{by} = e^{ik\Lambda m}\Big(B_+ e^{ik_{bz}(z-m\Lambda-d_c-d_a-d_g)} + \\ \qquad +B_- e^{-ik_{bz}(z-m\Lambda-d_c-d_a-d_g)}\Big) \\ H_{bx} = \frac{k_{bz}}{k_0} e^{ik\Lambda m}\Big(-B_+ e^{ik_{bz}(z-m\Lambda-d_c-d_a-d_g)} + \\ \qquad +B_- e^{-ik_{bz}(z-m\Lambda-d_c-d_a-d_g)}\Big). \end{cases} \quad (8)$$

The dispersion law for TPP can be obtained by stitching of Eq. (4) and Eq. (5), at the boundaries of $z=0$:

$$\begin{cases} M_- = G_+ + G_- \\ \frac{k_{mz}}{k_0} M_- = \frac{k_{gz}}{k_0}\Big(-G_+ + G_-\Big). \end{cases} \quad (9)$$

From Eq. (9), we find the ratio between G_+ and G_-:

$$G_+ = \frac{k_{gz}-k_{mz}}{k_{gz}+k_{mz}} G_- = r_{gm_\perp} G_-. \quad (10)$$

Let's stitch Eq. (5) and Eq. (6) at the boundary $z=d_g$:

$$\begin{cases} G_+ e^{ik_{gz}d_g} + G_- e^{-ik_{gz}d_g} = C_+ + C_- \\ \frac{k_{gz}}{k_{cz}}\Big(-G_+ e^{ik_{gz}d_g} + G_- e^{-ik_{gz}d_g}\Big) = C_- - C_+ \end{cases} \quad (11)$$

Similarly, stitching the equations on boundary $z=d_c+d_g$ taking into account $m\Lambda = 0$:

$$\begin{cases} C_+ e^{ik_{cz}d_c} + C_- e^{-ik_{cz}d_c} = e^{ik\Lambda}\Big(A_+ + A_-\Big) \\ \frac{k_{cz}}{k_{az}}\Big(-C_+ e^{ik_{cz}d_c} + C_- e^{-ik_{cz}d_c}\Big) = e^{ik\Lambda}\Big(-A_+ + A_-\Big). \end{cases} \quad (12)$$

On the boundary $z = m\Lambda + d_c + d_a + d_g$:

$$\begin{cases} A_+ e^{ik_{az}d_a} + A_- e^{-ik_{az}d_a} = \Big(B_+ + B_-\Big) \\ \frac{k_{az}}{k_{bz}}\Big(-A_+ e^{ik_{az}d_a} + A_- e^{-ik_{az}d_a}\Big) = \Big(-B_+ + B_-\Big). \end{cases} \quad (13)$$

Let's express G_- by adding up the equations from Eq. (11):

$$G_- = \frac{1}{2} e^{ik_{gz}d_g}\Big(C_+\big(1-\frac{k_{cz}}{k_{gz}}\big) + C_-\big(1+\frac{k_{cz}}{k_{gz}}\big)\Big) \quad (14)$$

Let's express G_+ by subtracting Eq. (11) system from each other:

$$G_+ = \frac{1}{2} e^{-ik_{gz}d_g}\Big(C_-\big(1-\frac{k_{cz}}{k_{gz}}\big) + C_+\big(1+\frac{k_{cz}}{k_{gz}}\big)\Big) \quad (15)$$

Similarly, we can express C_- by adding up the equations from Eq. (12):

$$C_- = \frac{A_+}{2} e^{ik\Lambda} e^{ik_{cz}d_c}\left(1-\frac{k_{az}}{k_{cz}}\right) + \\ + \frac{A_-}{2} e^{ik_{cz}d_z} e^{ik\Lambda}\left(1+\frac{k_{az}}{k_{cz}}\right), \quad (16)$$

then we can express C_+ by subtracting the equations from Eq. (12):

$$C_+ = \frac{A_+}{2} e^{-ik\Lambda} e^{-ik_{cz}d_c}\left(1+\frac{k_{az}}{k_{cz}}\right) + \\ + \frac{A_-}{2} e^{-ik_{cz}d_z} e^{ik\Lambda}\left(1-\frac{k_{az}}{k_{cz}}\right). \quad (17)$$

Substitute Eq. (14) and Eq. (15) into equation Eq. (10):

$$\begin{cases} C_-\Big(1-\frac{k_{cz}}{k_{gz}}\Big) + C_+\Big(1+\frac{k_{cz}}{k_{gz}}\Big) = \\ = r_{gm\perp} e^{2ik_{gz}d_g} C_+\Big(1-\frac{k_{cz}}{k_{gz}}\Big) + r_{gm\perp} e^{2ik_{gz}d_g} C_-\Big(1+\frac{k_{cz}}{k_{gz}}\Big). \end{cases} \quad (18)$$

Substitute Eq. (16) and Eq. (17) into equation Eq. (12):

$$\begin{cases} A_+ e^{-ik_{cz}d_c} - r_{mc\perp} r_{ac\perp} A_+ e^{ik_{cz}d_c} + \\ \qquad +A_- e^{-ik_{cz}d_c} r_{ac\perp} - A_- e^{ik_{cz}d_c} r_{mc\perp} = 0 \\ A_+(e^{-ik_{cz}d_c} - r_{mc\perp} r_{ac\perp} e^{ik_{cz}d_c}) + \\ \qquad +A_-(e^{-ik_{cz}d_c} r_{ac\perp} - e^{ik_{cz}d_c} r_{mc\perp}) = 0, \end{cases} \quad (19)$$

here $r_{ac\perp} = (k_{cz}-k_{az})/(k_{cz}+k_{az})$.

From Eq (18), we determine the coefficient connecting the amplitude C_+ and C_-:

$$C_+ = \frac{r_{gm\perp} e^{2ik_{gz}d_g} - r_{gc}}{1 - r_{gm\perp} r_{gc} e^{2ik_{gz}d_g}} C_- = JC_-. \quad (20)$$

From Eq (19) , we determine the coefficient connecting the amplitude A_+ and A_-

$$A_+ = \frac{e^{ik_{cz}d_c} J - e^{-ik_{cz}d_c} r_{ac\perp}}{e^{-ik_{cz}d_c} - J r_{ac\perp} e^{ik_{cz}d_c}} A_- = GA_-. \quad (21)$$

Stitching fields at the boundary of PhC layers can be written as a system:

$$\begin{cases} A_+\Big(e^{ik\Lambda} - e^{ik_{az}d_a} e^{ik_{bz}d_b}\Big) + \\ \qquad +r_{ab\perp} A_-\Big(e^{ik\Lambda} - e^{-ik_{az}d_a} e^{ik_{bz}d_b}\Big) = 0 \\ r_{ab\perp} A_+\Big(e^{ik\Lambda} - e^{ik_{az}d_a} e^{-ik_{bz}d_b}\Big) + \\ \qquad +A_-\Big(e^{ik\Lambda} - e^{-ik_{az}d_a} e^{-ik_{bz}d_b}\Big) = 0. \end{cases} \quad (22)$$

Substitute A_+ from Eq. (21) into the Eq (22):

$$\begin{cases} GA_-\left(e^{ik\Lambda} - e^{ik_{az}d_a}e^{ik_{bz}d_b}\right)+ \\ \qquad +r_{ab\perp}A_-\left(e^{ik\Lambda} - e^{-ik_{az}d_a}e^{ik_{bz}d_b}\right) = 0 \\ r_{ab\perp}GA_-\left(e^{ik\Lambda} - e^{ik_{az}d_a}e^{-ik_{bz}d_b}\right)+ \\ \qquad +A_-\left(e^{ik\Lambda} - e^{-ik_{az}d_a}e^{-ik_{bz}d_b}\right) = 0. \end{cases} \tag{23}$$

Let's express $e^{ik\Lambda}$ from the upper and lower equations of Eq (23) and equate:

$$\frac{Ge^{ik_{az}d_a}e^{ik_{bz}d_b} + r_{ab\perp}e^{-ik_{az}d_a}e^{ik_{bz}d_b}}{G + r_{ab\perp}} = \\ = \frac{r_{ab\perp}Ge^{ik_{az}d_a}e^{-ik_{bz}d_b} + e^{-ik_{az}d_a}e^{-ik_{bz}d_b}}{r_{ab\perp}G + 1}. \tag{24}$$

Let's reveal the proportion and give similar ones:

$$\begin{gathered} r_{ab\perp}G^2(e^{ik_{az}d_a}e^{ik_{bz}d_b} - e^{ik_{az}d_a}e^{-ik_{bz}d_b})+ \\ +r_{ab\perp}^2G(e^{-ik_{az}d_a}e^{ik_{bz}d_b} - e^{ik_{az}d_a}e^{-ik_{bz}d_b})+ \\ +G(e^{ik_{az}d_a}e^{ik_{bz}d_b} - e^{-ik_{az}d_a}e^{-ik_{bz}d_b})+ \\ +r_{ab\perp}(e^{-ik_{az}d_a}e^{ik_{bz}d_b} - e^{-ik_{az}d_a}e^{-ik_{bz}d_b}) = 0. \end{gathered} \tag{25}$$

Exponential differences can be represented as trigonometric functions. As a result, Eq (25) take the form:

$$\begin{gathered} r_{ab\perp}G^2e^{ik_{az}d_a}\sin(k_{bz}d_b) + r_{ab\perp}^2G\sin(k_{bz}d_b - k_{az}d_a)+ \\ G\sin(k_{bz}d_b + k_{az}d_a) + r_{ab\perp}e^{-ik_{az}d_a}\sin(k_{bz}d_b) = 0. \end{gathered} \tag{26}$$

The solution of the obtained dispersion law for TPP makes it possible to determine the wavelength and spectral width of the resonance line.

Notes and references

1 P. Yeh, *Journal of the Optical Society of America*, 1979, **69**, 742.