Supplementary Information (SI) for Nanoscale.

Nonlinear Memristor Model With Exact Solution Allows for ex-situ Reservoir Computing Training and in-situ Inference Supplemental Information

Nicholas Armendarez,[†] Md Sakib Hasan*,[‡] and Joseph S. Najem*,[†]

†*Department of Mechanical Engineering, The Pennsylvania State University, 336 Reber Building, University Park, PA 16802*

‡*Department of Electrical and Computer Engineering, The University of Mississippi, 310 Anderson Hall, University, MS 38677*

E-mail: mhasan5@olemiss.edu,jsn5211@psu.edu

Supporting Figures

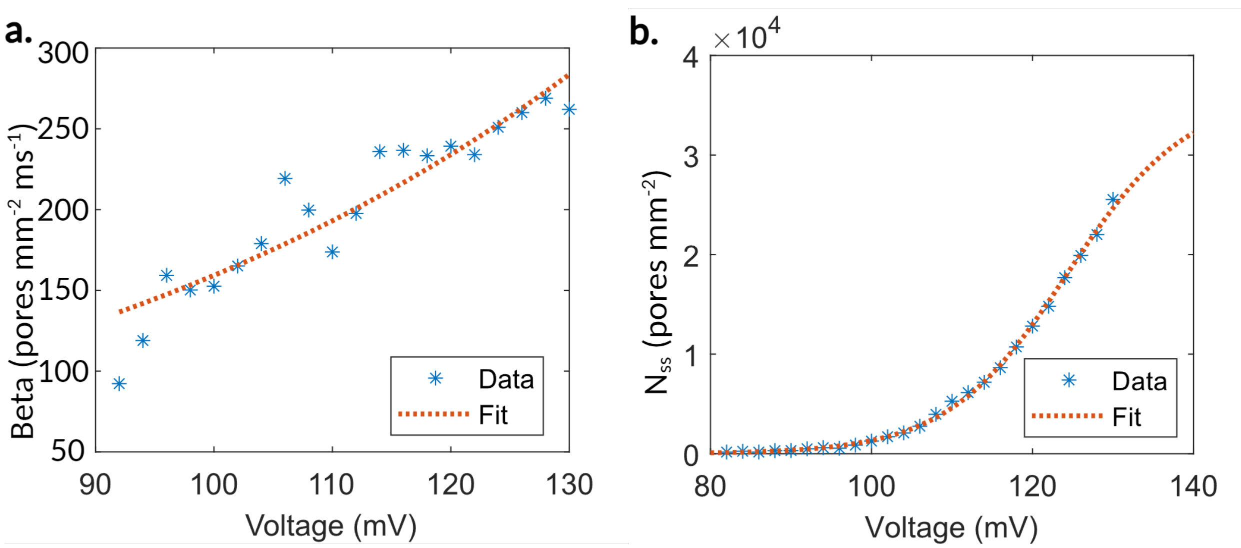

Figure S1: The fitted RDE parameters β and N_{ss} as a fit function of voltage. a. The β parameter, which represents the rate constant, is fit to an exponential function of voltage. b. The N_{ss} parameter, which represents the maximum pore density, is fit to a sigmoidal function of voltage.

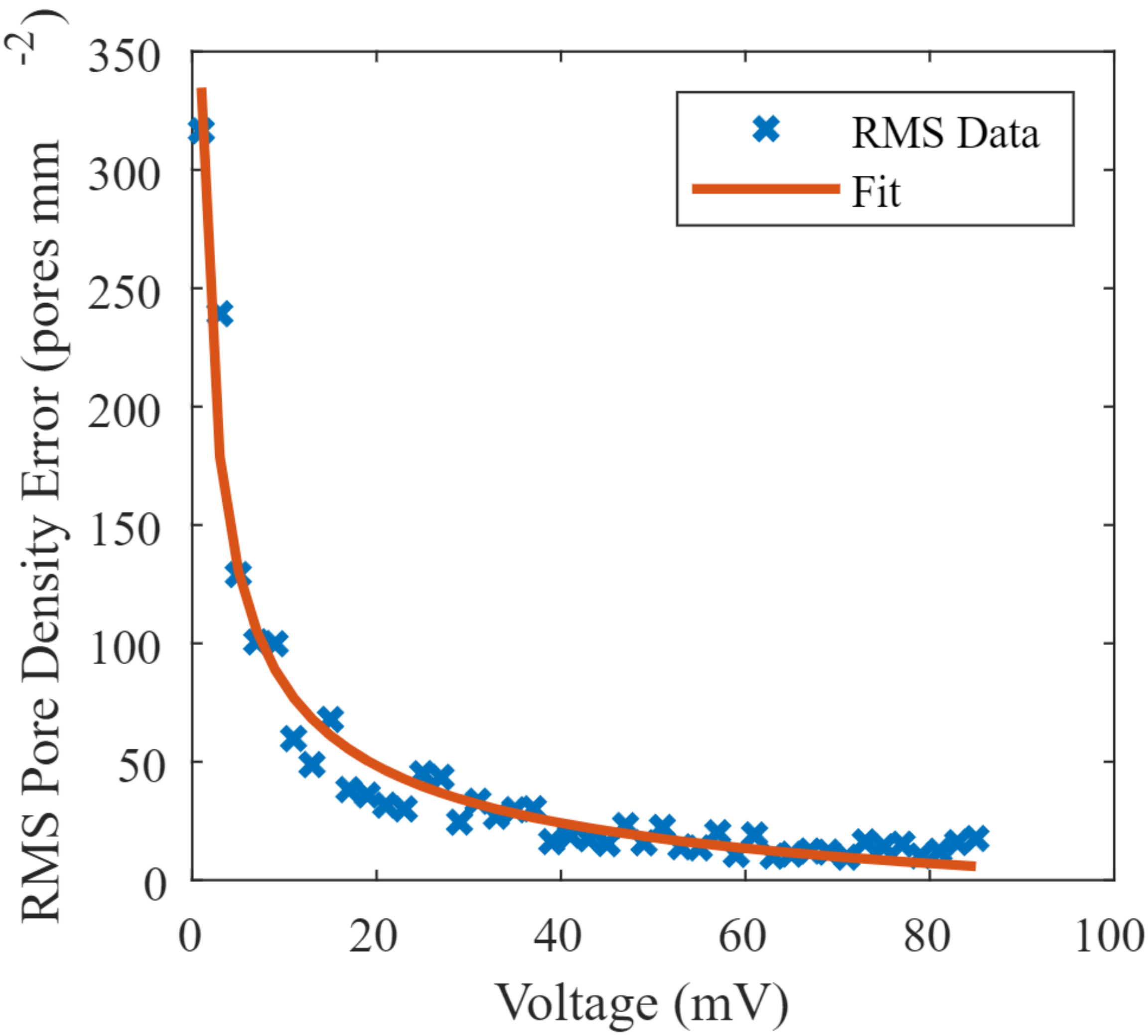

Figure S2: The RMS error in pore density as a function of applied voltage is fit to an inverse square root relation. The magnitude of added noise to the model is given by the fit equation as $RMS\ noise = \frac{A_n}{\sqrt{V}} + B_n$. Where A_n and B_n were found to be 368 and -34.2 pores per mm^2, respectively.

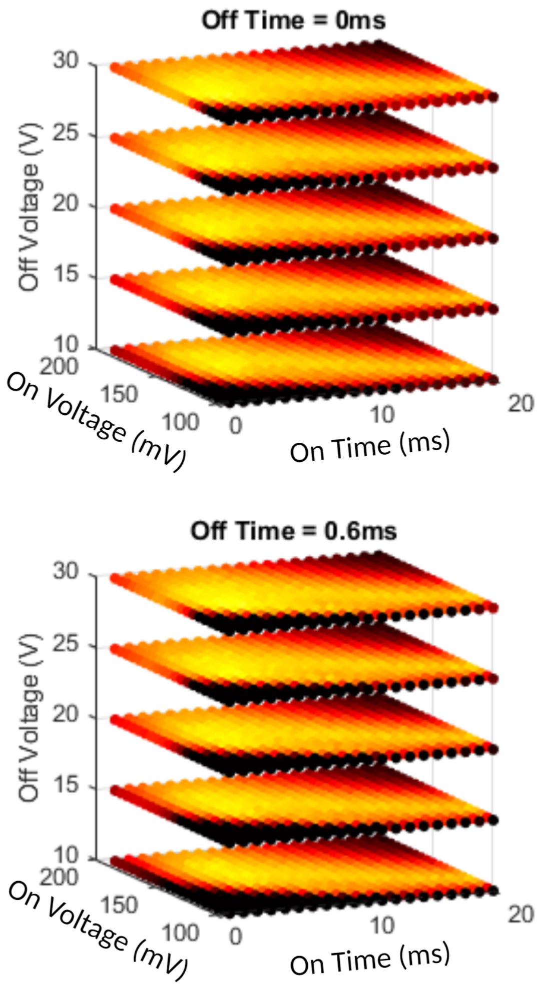

Figure S3: Expanded select slices of the 4D hyperparameter grid search. The three axes of each plot represent the input encoding parameters that caused the most variation in training response: period, on voltage, and off voltage. Each plot represents a different off time, the input encoding parameter that caused the least variation.

.

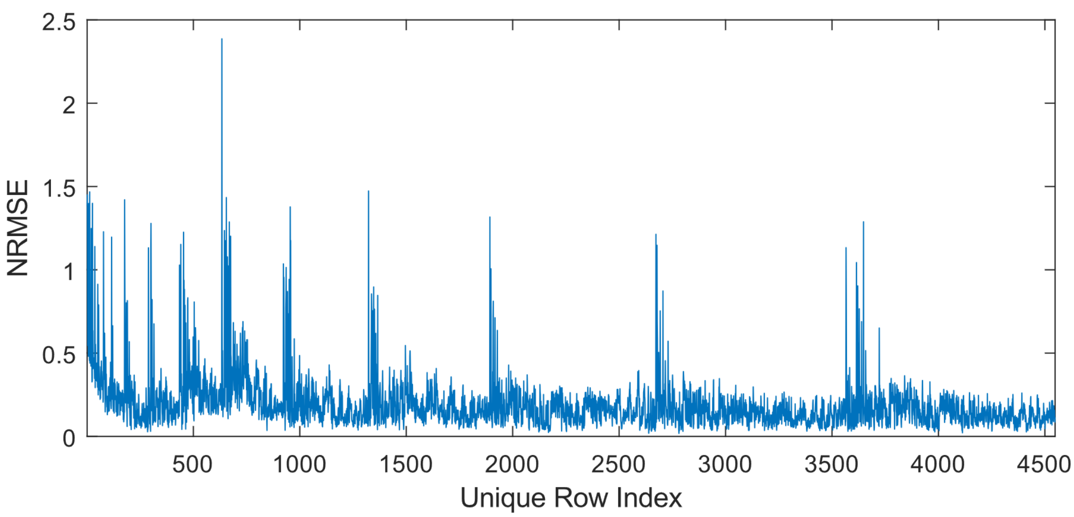

Figure S4: The RDE model was run for the 4548 unique pixel inputs (rows or columns of length 20 pixels) of the 1000 test data used in this study. The NRMSE between the simulated and experimental responses is shown. The overall average NRMSE of all samples was 0.189.

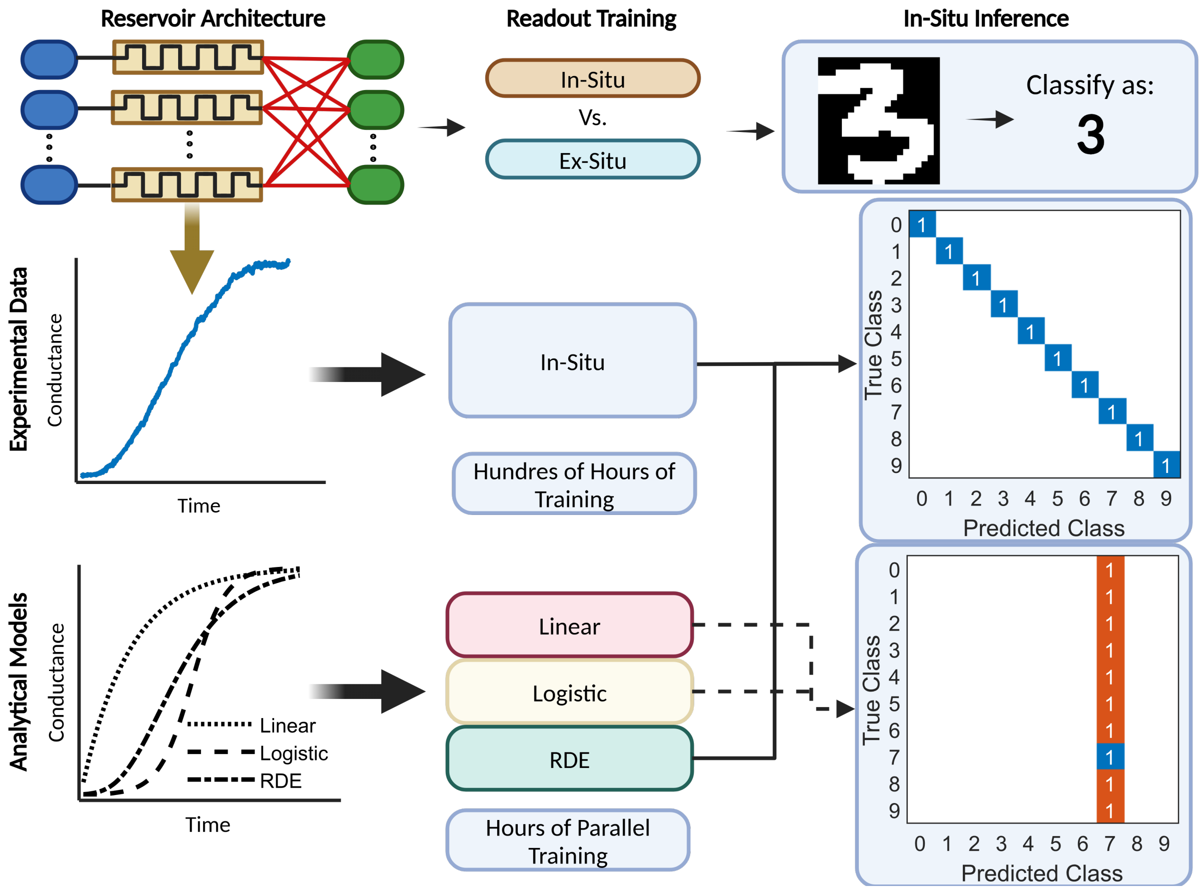

Figure S5: The figure shows a flowchart of the memristor reservoir training methods. The initial architecture is fixed but an option is provided to train readout weights in-situ in physical devices or ex-situ using a model. Finally, the physical in-situ reservoir is used for inference. Below, the two options are shown, and the modeling option is expanded upon to give various model possibilities. While the in-situ hyperparameter tuning would take hundreds of hours, the model-based training can be completed in a fraction of the time. Finally, we note the importance of the RDE model over alternative analytical models for accurate in-situ inference in the MNIST classification task.

Supporting Table

Table S1: This table lists all the RDE model-specific parameters that were fitted for in this study.

$\beta_0 1$	$\beta_0 2$	$V_{\beta 1}$	$V_{\beta 2}$	V_{Thresh}	N_{inf}	N_{α}	V_h	Z
ms^{-1}	ms^{-1}	mV^{-1}	mV^{-1}	mV	$pores\ mm^2$	mV^{-1}	mV	dimensionless
3.33e5	3.36e4	-450	17.0	107	3.609e4	96	139	0.00015

Supporting Note

The comparison of ex-situ to in-situ grid search times discussed in the results section involves two considerations. First, the ex-situ grid search time was measured directly via MATLAB's built-in timer function and was found to be 4.85 hours when run on a 48-core server node. The total time it would take to collect all training data in-situ was calculated by multiplying the number of unique rows to be run of the training data (24,145 for the MNIST task as described in the main text) by the number of pixels per row (20) and by the sum of the period and off time for a single pulse for each set of parameters tested. This calculation results in the in-situ grid search would take 11,751 hours to run on a single memristor training setup. Assuming a 40 parallel memristor setup capable of running the MNIST task as described in the main text, the total runtime remains 293 hours.