## Supplementary information for:

## Identifying the ideal thermodynamics of non-stoichiometric oxygen-carrier materials for chemical looping water-gas shift

## Derivation of dimensionless number $\lambda_0$

The dimensionless number  $\lambda_0$ , which is used extensively throughout the paper, is defined by the ratio of moles of usable oxygen in the solid phase  $N_{OCM, oxygen, usable}$ , divided by the moles of oxygen present in the passing gas phase stream during a half cycle  $N_{Gas, oxygen}$ .

$$\lambda_{O} = \frac{N_{OCM, oxygen, usable}}{N_{Gas, oxygen}}$$
Eq. S1

The moles of oxygen present in the passing gas phase stream in a half cycle is given by the following,

$$N_{Gas, oxygen} = F_0 t$$
 Eq. S2

where  $F_0$  refers to the molar flowrate of oxygen in the gas phase and t refers to the duration of the half cycle.

The usable moles of oxygen in the OCM is logically given by the following,

$$N_{OCM, oxygen, usable} = N_{OCM, oxygen, oxidation} - N_{OCM, oxygen, reduction}$$
 Eq. S3

where  $N_{OCM, oxygen, oxidation}$  and  $N_{OCM, oxygen, reduction}$  refer to the moles of oxygen in the OCM at the end of the oxidation and reduction half cycles respectively.

The moles of oxygen in the OCM at the end of the oxidation and reduction half cycles are given by,

$$N_{OCM, oxygen, oxidation} = (S - \delta_{ox})N_{OCM}$$
 Eq. 54

$$N_{OCM, oxygen, reduction} = (S - \delta_{red}) N_{OCM}$$
 Eq. 55

where  $N_{OCM}$  refers to the moles of OCM, S refers to the oxygen stoichiometry when  $\delta$  is equal to 0 (e.g. for the OCM La<sub>0.6</sub>Sr<sub>0.4</sub>FeO<sub>3- $\delta$ </sub> the value of S is equal to 3), and  $\delta_{red}$  and  $\delta_{ox}$  are

the degrees of non-stoichiometry of the OCM during steady cycling at the end of the reduction and oxidation half cycles respectively.

Subtracting Eq. S4 from Eq. S5 leads to the following equation:

$$N_{OCM, oxygen, usable} = (\delta_{red} - \delta_{ox})N_{OCM}$$
 Eq. S6

Substituting Eq. S2 and Eq. S6 into Eq. S1 therefore leads to the following equation for the dimensionless number  $\lambda_0$ :

$$\lambda_{O} = \frac{\left(\delta_{red} - \delta_{ox}\right)N_{OCM}}{F_{O}t}$$
Eq. S7

Verifying  $\lambda_0$  using different values of  $\delta_{red} - \delta_{ox}$ , t,  $N_{OCM}$  and  $F_O$ The simulation results in Table S1 demonstrate that the conversions are dependant only on the values of  $\log_{10} p_{O_2}{}^{mid}$ ,  $k_{grad}$  and  $\lambda_0$ . For example, a  $\lambda_0$  value of 0.5 gives  $X_{CO}$  and  $X_{H_2O}$  values of ~50% regardless of how  $\lambda_0$  is derived with changes to  $\delta_{red} - \delta_{ox}$  and  $N_{OCM}$ .

Table S1: Conversions determined from the simulation using different values of  $\lambda_0$  using various  $\delta_{red} - \delta_{ox}$ ,  $F_{O_2}$ , t and  $N_{OCM}$  values. The values of  $k_{grad}$  and  $\log_{10} p_{O_2mid}$  are kept constant at -1.15 and -17.8 respectively. The values of conversion that are shown are those during steady cycling when the difference between  $X_{co}$  and  $X_{H_2O}$  is < 0.01.

λο	$\delta_{red} - \delta_{ox}$	$F_{O}$ (mol min <sup>-1</sup> )	<sup>t</sup> (s)	<sup>N</sup> <sub>OCM</sub> (mols)	$X_{\rm H_{2}O}$ , $X_{\rm CO}$
1	0.0225	1.7361 × 10 <sup>-6</sup>	60	0.004639	0.94
1	0.0225	8.6806 × 10 <sup>-7</sup>	120	0.004639	0.94
1	0.0225	1.7361 × 10⁻6	120	0.009278	0.94
1	0.0449	3.4722 × 10⁻ <sup>6</sup>	60	0.004639	0.94
0.5	0.0112	1.7361 × 10⁻6	60	0.004639	0.50
0.5	0.0112	8.6806 × 10 <sup>-7</sup>	120	0.004639	0.50
0.5	0.0112	1.7361 × 10 <sup>-6</sup>	120	0.009278	0.50
0.5	0.0225	3.4722 × 10 <sup>-6</sup>	60	0.004639	0.50

Fig. S1a and Fig. S1d show that there are no noticeable changes to the shapes of the outlet concentration profiles at  $\lambda_0 = 1$  when using different  $F_0$  and  $\delta_{red} - \delta_{ox}$  values. Meanwhile the outlet concentration profile at 60s feed durations compared to 120 s feed durations in

Fig. S1a and Fig. S1d compared to Fig. S1b and Fig. S1c is a half-scaled version of the 120 s profile along the x-axis. This indicates that as expected while a greater volume of  $H_2$  is



possible when the value of  $\delta_{red} - \delta_{ox}$  is greater or more moles of OCM are used, the conversions and outlet product quality is still determined by the value of  $\lambda_0$  and the specific  $\log_{10} p_{O_2 mid}$  and  $k_{grad}$  of the  $\delta^{-p_{O_2}}$  curve.

Figure S1: Comparison on the effect on outlet concentration profiles for  $\lambda_0 = 1$  when using a)  $\delta_{red} - \delta_{ox} = 0.0225$ ,  $F_0 = 1.7361 \times 10^{-6}$  mol min<sup>-1</sup>, t = 60s,  $N_{OCM} = 0.004639$  mols, b)  $\delta_{red} - \delta_{ox} = 0.0225$ ,  $F_0 = 8.6806 \times 10^{-7}$  mol min<sup>-1</sup>, t = 120s,  $N_{OCM} = 0.004639$  mols, c)  $\delta_{red} - \delta_{ox} = 0.0225$ ,  $F_0 = 1.7361 \times 10^{-6}$  mol min<sup>-1</sup>, t = 120s,  $N_{OCM} = 0.004639$  mols, c)  $\delta_{red} - \delta_{ox} = 0.0225$ ,  $F_0 = 1.7361 \times 10^{-6}$  mol min<sup>-1</sup>, t = 120s,  $N_{OCM} = 0.004639$  mols, c)  $\delta_{red} - \delta_{ox} = 0.0225$ ,  $F_0 = 1.7361 \times 10^{-6}$  mol min<sup>-1</sup>, t = 120s,  $N_{OCM} = 0.004639$  mols. The values of  $k_{grad}$  and  $\delta_{red} - \delta_{ox} = 0.0449$ ,  $F_0 = 3.4722 \times 10^{-6}$  mol min<sup>-1</sup>, t = 60s,  $N_{OCM} = 0.004639$  mols. The values of  $k_{grad}$  and  $\log_{10} p_{0_2}$  are kept constant at -1.15 and -17.8 respectively. The profiles are taken Fig. S2 shows the outlet concentration profiles corresponding to selected  $\lambda_0$  values for the optimal  $\delta^{-\frac{p}{2}}$  relationship. As expected,  $\lambda_0 < 1$  results in less available oxygen capacity and gives a decrease in conversions and product quality. For  $\lambda_0 <= 0.1$  significant quantities of unreacted feed gas start to be seen in the outlet concentration profiles. At  $\lambda_0 = 1 \times 10^{-3}$  it is seen that the inlet streams pass through the reactor almost unreacted with unacceptable product quality, while at a  $\lambda_0$  value of 1000 almost pure product H<sub>2</sub> and CO<sub>2</sub> can be seen

only limited by the  $p_{0_2}$  of the inlet feeds. It would still be possible to achieve high conversions at low  $\lambda_0$  values as the gradient of the curve  $k_{\text{grad}}$  and  $\log_{10} p_{0_2 \text{mid}}$  are not changed, however a greater mass of OCM would need to be used to be to be able to do so which would be more expensive industrially. Alternatively an OCM with a similar  $\delta_{p_0}^{p_0}$ relationship but with a larger maximum degree of oxygen non-stoichiometry could be used if it existed.



Figure S2: Outlet concentration profiles determined from the simulation using different values for  $\lambda_0$  of the  $\delta$ - $p_{0_2}$  plot at 1093 K. The values of  $k_{\text{grad}}$  and  $\log_{10} p_{0_2 mid}$  are kept constant at -1.15 and -17.8 respectively. The profiles are taken during steady cycling when the difference between  $X_{\text{co}}$  and  $X_{\text{H},0}$  is < 0.01.