Electronic Supplementary Material (ESI) for Soft Matter.
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Supplementary Information: Cutting Soft Matter: Scaling relations controlled by toughness, friction, and wear[†]

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1 Experimental Measurements

1.1 Wire cutting experiments

Sample preparation, for cutting experiments, involves gluing the bottom of the hydrogels on a flat substrate fixed to the bottom clamp of the tensile testing machine (Instron, model 68SC-1), while the blade is a metal wire fixed onto the top clamp. The wire indents the sample moving orthogonally to its top surface, where the cut is initiated and then propagated. The wire moves at a constant prescribed velocity, while the force is recorded with a load cell (500 N).

1.2 Tensile tests of pristine samples: Evaluation of μ and W_f

Tensile rupture tests were performed on pristine "dog-bone" samples shown in Fig. S1. In the same figure, we also report the plot of the engineering stress (force over initial area) versus stretch (length over initial length) during the test. The plot reports a critical stress of $\sigma_c = 3.1\ kPa$. The area underneath the curve gives $W_f = 6.973 \pm 0.5\ kJ/m^3$ the critical strain energy density at failure. The stress-stretch plot was fitted with a neo-Hookean hyperelastic model prediction with an accuracy of $R^2 = 0.989$, unraveling a shear modulus of $\mu = 685 \pm 30\ Pa$.

Tensile test hydrogel samples are cut into dog-bone-shaped specimens (gauge length 30 mm, width 5 mm, and thickness 0.5 mm) and glued (Krazy Glue) to PET clamps. Samples are subject to uniaxial stretch by a tensile testing machine (Instron model 68SC-1) at a constant prescribed velocity (0.2 mm/s) while the force is recorded with a load cell (500 N). The engineering stress is obtained by dividing the force by the initial area of the sample cross-section.

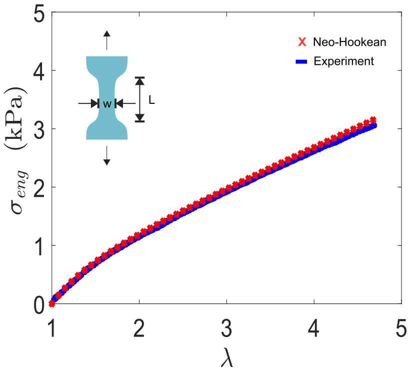

Fig. S1 Plot of engineering stress (force over the initial area) Vs stretch (length over initial length) for the uniaxial tensile test of a pristine sample of polyacrylamide (PAAm) hydrogel.

1.3 Pure shear fracture tests: Evaluation of Γ

Pure shear testing[1] is done using two samples of identical dimensions L = 45 mm, T = 1.4 mm, H = 5 mm, where one is pristine, and the other one has a notch of length c = 17.5 mm along the length. The pristine sample is pulled at a large stretch, while the notched one is pulled to its critical failure stretch prompting fracture. The critical strain energy density of the pristine sample at the critical fracture stretch of the notched one is W_{ps} and is given by the area underneath the engineering stress versus stretch. We then have $\Gamma = W_{ps}H$, with H the undeformed height of the sample. The estimated value for fracture toughness is $\Gamma = 2.6 \pm 0.2\ J/m^2$. Fig. S2 reports the engineering stress versus stretch for both the pristine sample (red line) and the notched one (blue line).

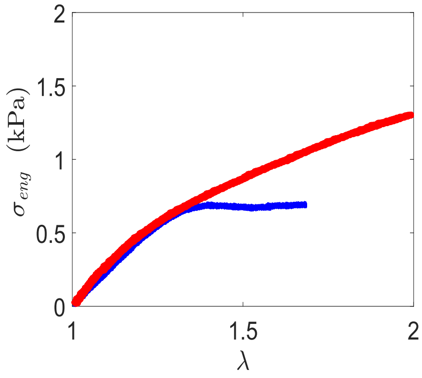

Fig. S2 Plot of engineering stress vs stretch for pure shear samples. Unotched sample (in red) and Notched sample (in blue)

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2 Finite Element Analysis

The finite element analysis (FEA) was used to evaluate the J-integral and the cutting force contribution F_D from wear and friction. From Eq. (7) we have $F_d/T = 2R_w \int_o^\pi \tau_c \, sin\theta d\theta$, which, from Eq. (8) can be decomposed into

$$\frac{F_d}{2TR_w} = \tau_w \int_{\theta_1}^{\theta_2} sin\theta \, d\theta + \xi \int_{\theta_1}^{\theta_2} P \, sin\theta \, d\theta \tag{S1}$$

where θ_1 and θ_2 are the angles at which we have zero contact pressure ($P(\theta_1) = P(\theta_2) = 0$). By rearranging Eq. (S1) we have

$$\frac{F_d}{2TR_w} = \tau_w \left(cos\theta_1 - cos\theta_2 \right) + \xi \frac{F_v}{TR_w} \tag{S2}$$

where $F_v = T \int_{\theta_1}^{\theta_2} P \, sin\theta \, R_w d\theta$ is the vertical force applied to the wire to push it up into the half-specimen, as shown in Fig. 2(b).

Using FEA, we calculate θ_1, θ_2, and F_v, as functions of the configuration of the wire-specimen system, which is given by the ratio R_w/δ. The simulation was performed on ABAQUS using R2D2 (a 2-node 2-D linear rigid link) for the cutting wire and CPE4H (a 4-node bilinear plane strain quadrilateral, hybrid, constant pressure) element to handle material incompressibility of the sample. Crack was assigned on the symmetry plane as an Engineering Feature with a crack extension direction (q-vector) to estimate the energy release rate.

Notes and references

1 R. Rivlin and A. G. Thomas, *Journal of polymer science*, 1953, **10**, 291–318.