Supplementary Information: Foam coarsening in a yield stress fluid

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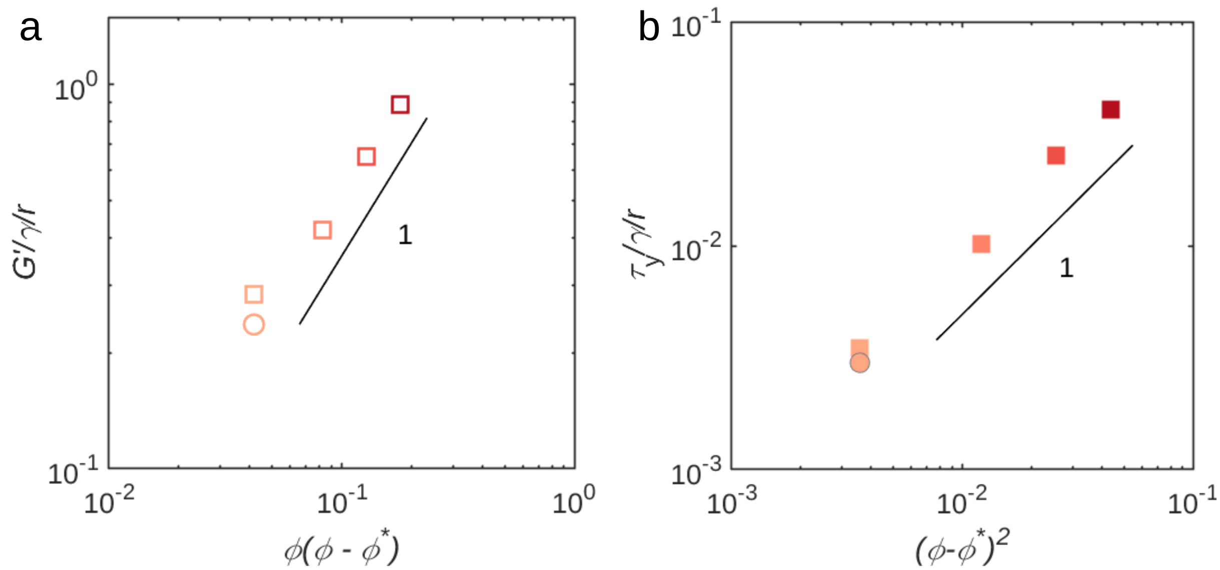

FIG. 1. Rheological properties of the emulsions. a: Evolution of the normalised storage modulus $G'/\gamma/r$, where γ is the surface tension and r is the oil droplets radius, with $\phi(\phi-\phi^\star)$ ($\phi^\star$ being the the random close packing fraction for monodisperse hard spheres and is close to 64 %) as proposed by [35]. b: Evolution of the normalised yield stress $\tau_y/\gamma/r$ with $(\phi-\phi^\star)^2$ as proposed by Mason [35]. Squares refer to samples made with rapeseed oil and circles to sunflower oil samples.

There are different ways to quantify the change in the distributions shape along the coarsening, characterising the progressive accumulation of smaller bubbles within the foamed emulsion. One is through the evolution of the radius of the bubble size distribution peak R_{mode} with the mean radius $\langle R\rangle$. For an aqueous foam in the scaling state, R_{mode} should scale linearly with $\langle R\rangle$ (pale blue crosses in Fig. 2) , but in a foamed emulsion this is no longer true and R_{mode} starts to decrease at a certain time (pale orange circles in the same figure). This moment indicates the average radius at which the peak of the bubble size distribution is shifted towards smaller bubbles.

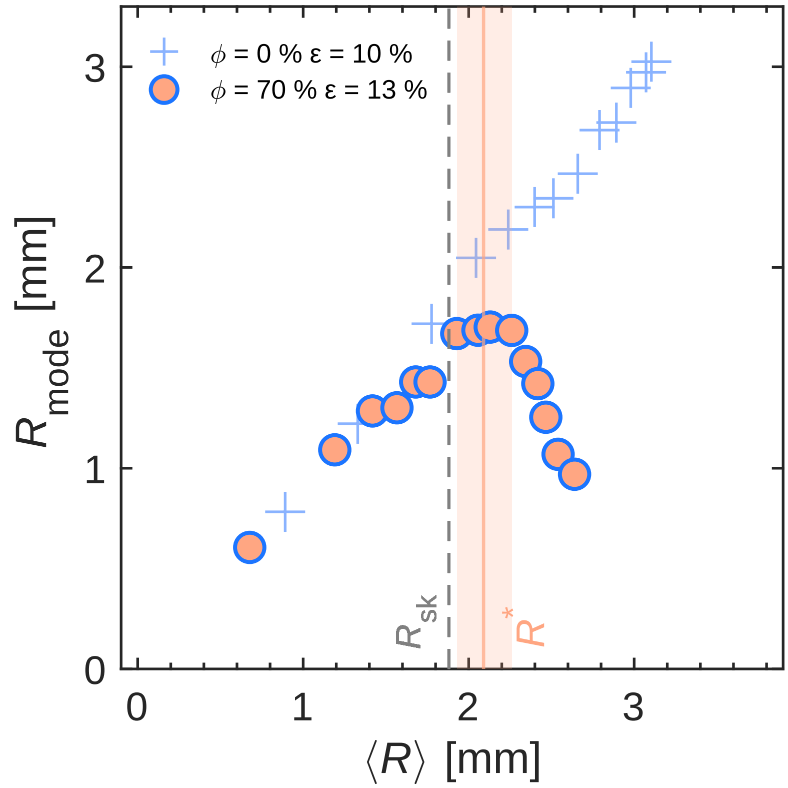

FIG. 2. Evolution of the radius of the bubble size distribution peak R_{mode} with the mean radius $\langle R \rangle$ for an aqueous foam with $\varepsilon = 10$ % (pale blue crosses) and a foamed emulsion with $\phi = 70$ % and $\varepsilon = 13$ % (pale orange and blue circles). The pale orange solid line gives the value of the critical average radius $R^\star$ at which the bubble size distribution shifts left, so towards smaller bubbles (according to the method described in section 3.2 of the main text). The shaded beige rectangle gives an idea of the uncertainty on this value. The grey dashed line gives the value of R_{sk} defined in section 3.3 of the main text. Both radii (R^* and R_{sk}) are similar and we conclude that both methods give reasonably close values of the threshold radius at which small bubbles start to accumulate.

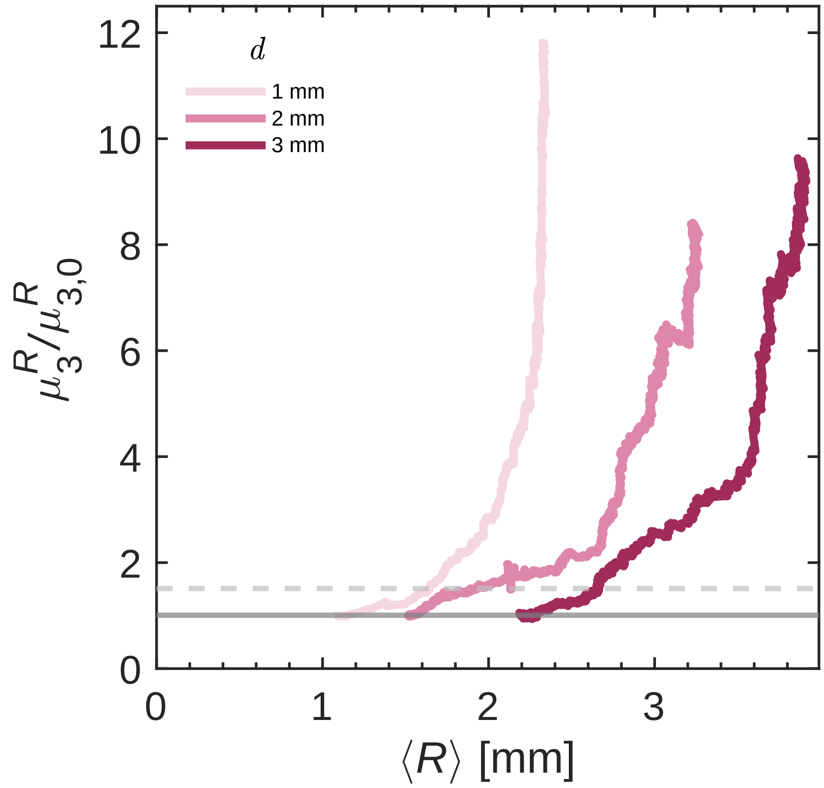

FIG. 3. Normalised third moment of the bubble radius distributions $\mu_3^R/\mu_{3,0}^R$ (with $\mu_{3,0}^R = \mu_3^R(t_0)$) plotted as a function of the average bubble size $\langle R \rangle$. The evolution is shown for a foamed emulsion with $\phi = 80$ % and $\varepsilon = 10$ % varying the confining gap from $d = 1$ mm (pale pink) to $d = 3$ mm (wine-coloured). The solid grey line shows the initial average plateau at early times. The dashed grey line indicates 150 % of this value; $R_{\rm sk}$ is found by the intersection between the curves and the dashed line. $R_{\rm sk}$ increases as the foamed emulsion is less confined, meaning that we observe changes in the bubble size distributions and in the foam structure at a larger radius as the gap d increases.

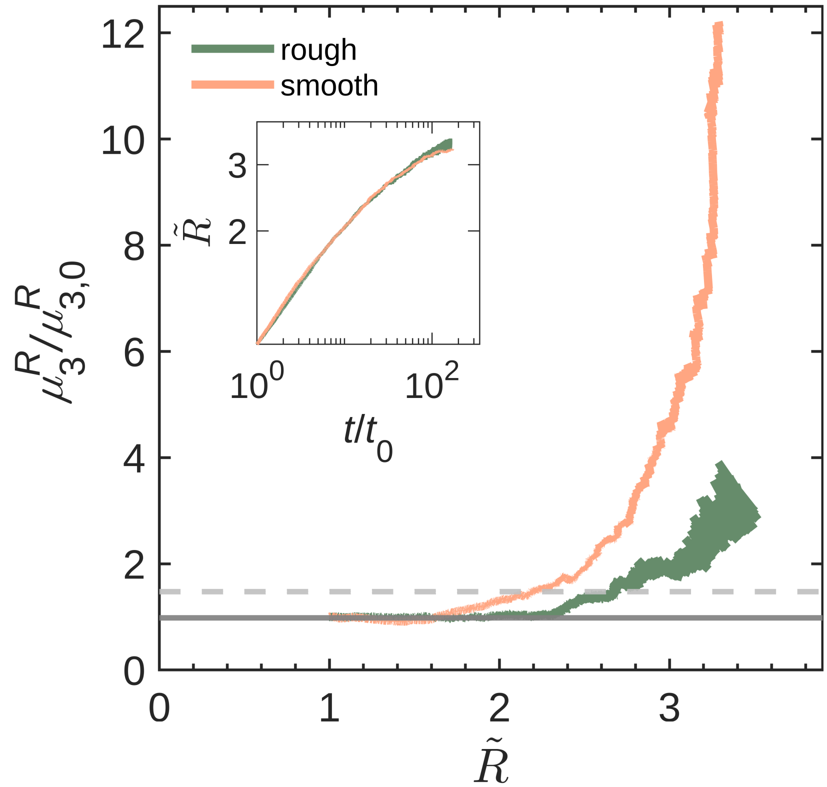

FIG. 4. Normalised third moment of the bubble radius distributions $\mu_3^R/\mu_{3,0}^R$ plotted as a function of the normalised average bubble size $\tilde{R} = \langle R\rangle/\langle R_0\rangle$. The inset shows the evolution of $\tilde{R}$ with normalised time t/t_0. The sample studied is a foamed emulsion with $\phi = 70$ % and $\varepsilon = 13$ %. The experiment is made either with smooth plates (pale orange curve) or with roughened surfaces (green curve) (see section 2.4 of the main text). The solid grey line shows the initial average plateau at early times. The dashed grey line indicates 150 % of this value; $R_{\rm sk}$ is found by the intersection between the curves and the dashed line. The initial average radius $\langle R_0\rangle$ is 0.84 mm for the smooth surfaces and 0.78 mm for the rough plates. Both experiments give similar values of $R_{\rm sk}$ (respectively 1.83 and 2.09 mm) given the experimental uncertainty. This is confirmed by Fig. 5 of SI.

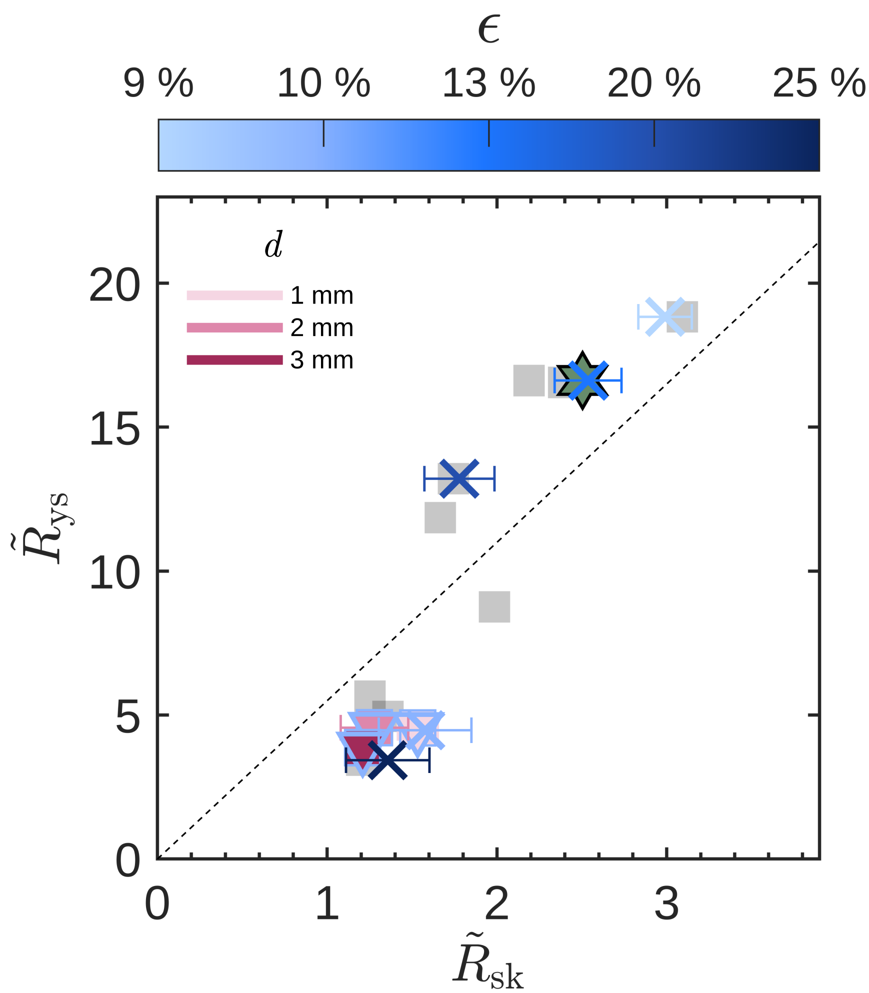

FIG. 5. Normalised theoretical critical radius $\tilde{R}_{\mathrm{ys}} = R_{\mathrm{ys}}/\langle R_0 \rangle$ *versus* different normalised critical radii indicating when the small bubbles experimentally start to accumulate. The grey points correspond to the $\tilde{R}_{\mathrm{sk}} = R_{\mathrm{sk}}/\langle R_0 \rangle$ shown in Fig. 7 of the main text. The reverse triangles are $\tilde{R}_{\mathrm{sk}}$ varying the confining gap from 1 (pale pink) to 3 mm (wine-coloured), for a foamed emulsion at 80 % of oil (cf. Fig. 3 SI). The green star with black contour corresponds to $\tilde{R}_{\mathrm{sk}}$ obtained with rough plates, to confirm the hypothesis of no slippage of section 3.4 (cf. Fig. 4 SI). The crosses correspond to the critical radius obtained by comparing the radius of the distribution peak with the mean radius $\langle R \rangle$ (cf. Fig. 4 SI and section 3.2 main text). The blue colour bar indicates the liquid fraction of the samples, from 9 % (pale blue) to 25 % (navy blue).