Supplementary Information – Torque about electrostatically charged spheres, changes how they stick

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Details of the charge measurement:

For the experiments in figure 2 all beads used had a diameter of 10mm. Upon purchase, the acrylic and silicone spheres already contained a 1.5mm diameter hole through the centre of the sphere. The PTFE, Polypropylene and Delrin spheres were held in a lathe and then drilled through to create one. The smaller beads used in figure 1 were made from PTFE and polypropylene.

Beads were washed in Methanol, detergent and deionised water. Each large bead was individually picked up on the end of insulated / discharged tweezers and dried with N_2. The acrylic beads, which were painted, were not washed in Methanol as this dissolved the paint. The glass container was washed with Acetone and then detergent and deionised water and also dried with N_2. Experiments were also done with variations on this protocol, including changing the order of solvents (Methanol last) or drying in a vacuum oven. However, this had no discernible effect on the results. The smaller beads were cleaned in a similar way however they were left to dry naturally in a covered container.

The charge measurement apparatus consists of a Perspex box which is gently purged with dry air resulting in a stable humidity ~ 25%. The box contains two large aluminium plates (120x120mm) mounted on translation stages. The plates are connected to a high voltage power supply (0-10kV) and for charge measurements are separated by 130mm.

The beads are suspended from a tag which was cut using a CO2 laser cutter from high density polystyrene sheet (0.75mm thick). This was designed so that when tweezers are inserted into the rectangular holes and compressed, the thin legs can be inserted into the hole in the bead. When the pressure is released these legs spring open exerting sufficient outwards pressure on the hole to grip the bead. The tag is attached to a 100μm diameter, L=1m long fishing line which is highly fluorinated. The other end of the wire is attached to the middle of a cylindrical puck which is located inside a 50mm diameter ring bearing. This arrangement enables the orientation of the tag relative to the electric field to be easily adjusted.

Measuring the deflection and rotation of the bead:

Images were taken using a camera (Ximea, MQ013CG-ON) positioned at right angles to the surface normal of the metal plates. These images were analysed using python and OpenCV to measure both the deflection Δx and rotation $\Delta\theta$ of the bead under application of an applied electric field.

The known diameter of the bead was used to calibrate the scale of the image. Thresholding and contour finding were used to find the bead and the tag within the image. For the tag, the projected width W was extracted from the image. The tag has a width L=7.5mm and a thickness t=0.75mm both of which contribute to the projected width depending on the angle of the tag.

$W = |L*\cos(\theta)|+|t*\sin(\theta)|$

Which allow one to extract the changes in angle. It should be noted that this technique is accurate when the tag, from the viewpoint of the camera, is at a significant angle / end on. However, when the tag is face on (ie L ~ W), one pixel change in the width can correspond to a significant change in the angle (~10°). To indicate this estimated error bars are added to a few representative points in figure 3. This has little effect on the measurement of the dipole since usually one is fitting a range of angles. However, where the dipole is small and the tag is face on, it becomes difficult to measure accurately. We therefore did not include a small number of measurements which had a total angular range +/-20°.

Capacitor configuration:

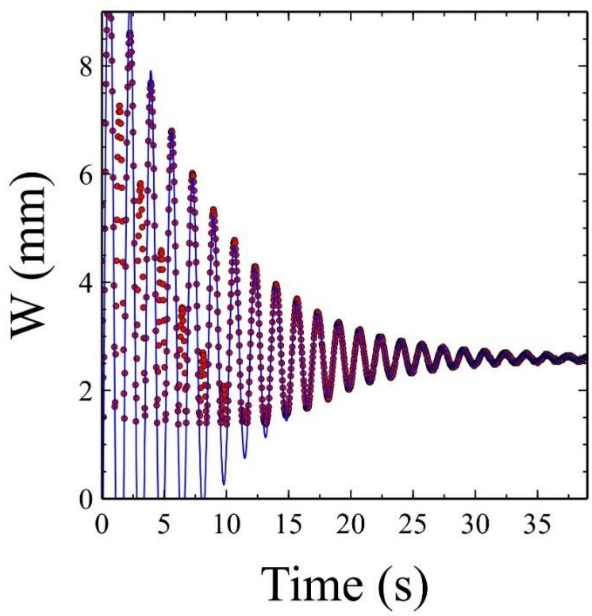
The distance between the metal plates is chosen to try and balance two competing sources of error:

1. If the plates are too far apart the approximation of a uniform electric field is poor.
2. If the plates are too close, there is a strong electric field generated by the mirror charge of the bead in the plates.

We aim to minimise the effects of the first by positioning the bead at the midpoint in all directions of the plates when the electric field is zero. This means that there are two mirror charge effects, one in the positive plate and one in the negative plate that are equal and opposite and hence cancel out when E=0. As the field is increased the bead only moves distances ~ a bead diameter. The data in figure 2 shows a very slight curve which is the result of this mirror charge. Since adding the correction to the analysis results in changes smaller than our typical variation between individual beads we have not included it.

Determining the torsional spring constant of the wire (K):

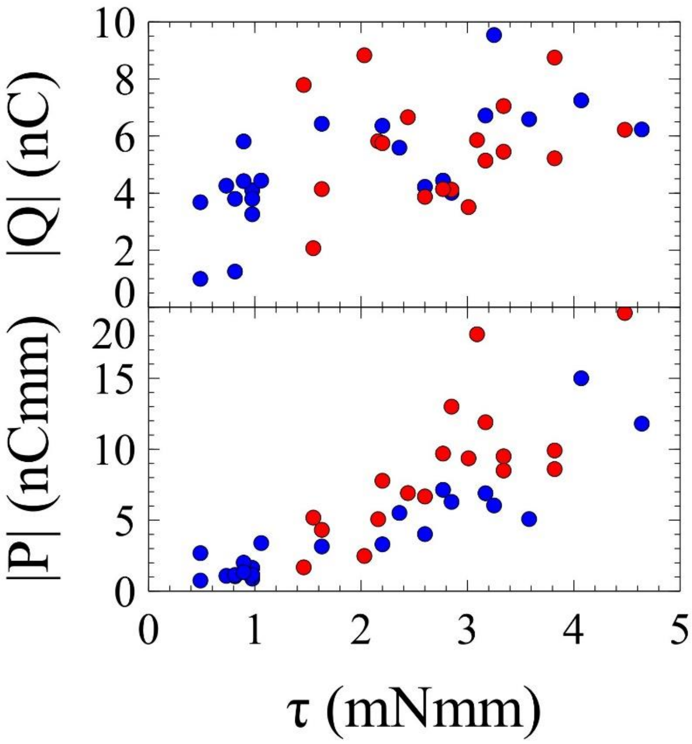
An uncharged bead was suspended from the wire. Rotational oscillations were excited in the wire by moving the point of attachment. The oscillatory motion of the bead was then filmed and found to undergo simple harmonic motion from which we extract the period of the oscillations to deduce the torsional spring constant K (supplementary figure 1). The moment of inertia of the system included the tag, although this contributed only a 5% correction. The torsional spring constant of the wire was measured to be $K=2.6x10^{-8}kgm^{2}s^{-2}$.



Supplementary Figure 1 - Torsional Spring Constant Measurement – rotational oscillations, excited in the bead, enable one to measure the torsional spring constant (K) of the wire. The projected width, W, of the tag obeys simple harmonic motion, allowing one to extract the resonant frequency $\omega=(K/I)^{0.5}$. Where I is the moment of inertia of the bead and tag.

Correlations between measured charge and dipole:

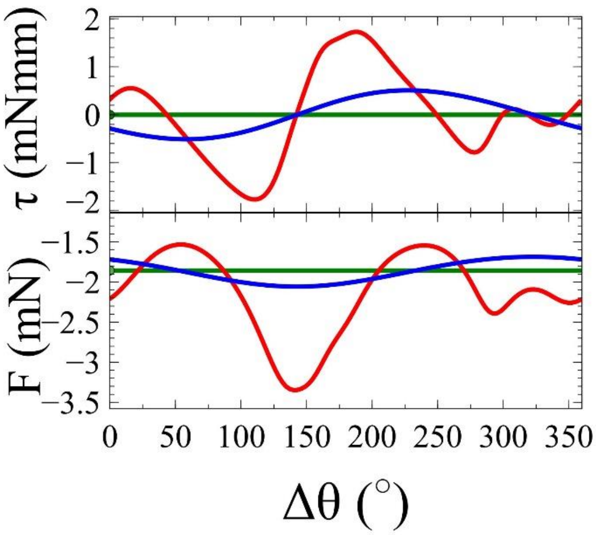
Measurements of the charge and dipole are obtained in a uniform field. Following each measurement the capacitor plates are discharged. One of the plates is moved closer to the bead which results in the bead coming into contact due to an attraction with its own mirror charge. A torque measurement (See figure 3) is then obtained by twisting the suspending fibre. This has very little correlation with the charge on the bead but is well correlated with the measured dipole moment. To aid comparison we plot the modulus of the charge and dipole moment. PTFE beads charge negatively whilst the Polypropylene particles charge positively.



Supplementary figure 2 - Correlations between measured torque and the charge (Q) and the dipole moment (P) for PTFE (●) and PP(●) particles. The dipole moment shows a much clearer correlation.

Comparing the expected torque and normal force for inhomogeneously charged particles

Our model calculates the pairwise coulomb interaction between all charges, both real and image. PTFE and PP have dielectric constants ~ 2 resulting in low polarizability. Finite element models suggest that under these conditions the correction due to induced polarization is very small [Feng2000] The model therefore does not include forces due to induced polarization.



Supplementary figure 3 – Additional example of torque and force curves produced by our model for a Q=5nC, N=20 charge sphere interacting with its image charges. Plots show a central charge model (green), the dipole model (blue) and the full charge distribution (red). The plots illustrate that with certain charge distributions the dipole model does not approximate the behaviour of the full distribution very well. The example here predicts the wrong sign for the torque at low angles and significantly underestimates the magnitude of the force.

Description of Supplementary videos

Supplementary video 1 : Video showing 4mm diameter polypropylene bead atop a glass coverslip on an Aluminium block. The angle of the surface is slowly increased using a DC motor. The video is filmed and played back at 30fps.

Supplementary video 2 : Dynamics of a charged 10mm diameter Polypropylene bead as it interacts with its own image charge in an Aluminium plate. The Aluminium plate is covered with glass coverslips. As the plate is moved inwards the bead is attracted and rotates, finally sticking to the glass. The original footage is filmed at 180fps and is played back at 50fps.

Supplementary video 3 : Measuring the torque due to an inhomogeneous charge distribution on a 10mm diameter Polypropylene bead as it interacts with its own image charge. The bead makes contact with glass coverslips that cover the Aluminium plate. The original footage is filmed at 50fps and played at 30fps.

Supplementary video 4 : Animation of the torque / force model of N randomly distributed charges interacting with their own image charges.

[Feng2000] J.Q. Feng *“Electrostatic interaction between two charged dielectric spheres in contact”* Phys. Rev. E 62, 2891-2897 (2000).