Supplementary Information (SI) for Soft Matter.

Supplementary Information

Noise level estimation and linear error propagation

The raw data of a strain chirp and its response may be expressed a stress timeseries $\sigma(t)$ and a stress timeseries $\varepsilon(t)$. The Fourier transform of these, $\sigma^*(\omega) = \sigma' + i\sigma''$ and $\varepsilon^*(\omega) = \varepsilon' + i\varepsilon'$ have a magnitude $|\sigma^*|$ and $|\varepsilon^*|$ respectively that follow a "pink" $1/f^\alpha$ power law spectrum between the lowest and highest requested frequencies ω_1 and ω_2. Between ω_2 and the Nyquist frequency ω_N, any measured signal can be assumed to originate in noise or interference. Assuming additive white Gaussian noise, the noise power can be estimated from this region:

$$N_{0,\sigma} = \frac{1}{\omega_N - \omega_2}\int_{\omega_2}^{\omega_N} |\sigma^*|^2 d\omega$$

and similarly for ε^*. The square root of N_0 can serve to estimate of the uncertainty at each frequency, $\Delta\sigma$ and $\Delta\varepsilon$. In practice, additional resonant noise and interference spikes appear throughout the spectrum, and so we use the median noise amplitude as a more robust estimator.

The complex stress σ^* and strain ε^* can be used to calculate the complex modulus through straightforward complex division:

$$E^* = \frac{\sigma^*}{\varepsilon^*} = \frac{\sigma' + i\sigma''}{\varepsilon' + i\varepsilon'}$$

$$E' = \frac{\sigma'\varepsilon' + \sigma''\varepsilon''}{|\varepsilon^*|^2}$$

$$E'' = \frac{\sigma''\varepsilon' - \sigma'\varepsilon''}{|\varepsilon^*|^2}$$

Our goal is to find the uncertainty in the real and imaginary components of the modulus, given our estimated uncertainty in the inputs, $\Delta\sigma$ and $\Delta\varepsilon$. Assuming linear, Gaussian, independent errors, linear error propagation can be applied:

$$\Delta f^2 = \sum_i \left(\frac{\partial f}{\partial x_i}\Delta x_i\right)^2$$

Assuming $\Delta\sigma' = \Delta\sigma'' = \Delta\sigma,\ \Delta\varepsilon' = \Delta\varepsilon'' = \Delta\varepsilon$:

$$\Delta E'^2 = \left(\frac{\partial E'}{\partial\sigma'}\Delta\sigma\right)^2 + \left(\frac{\partial E'}{\partial\sigma''}\Delta\sigma\right)^2 + \left(\frac{\partial E'}{\partial\varepsilon'}\Delta\varepsilon\right)^2 + \left(\frac{\partial E'}{\partial\varepsilon''}\Delta\varepsilon\right)^2$$

and similarly for E''. Taking partial derivatives and substituting, we obtain

$$\Delta E^{'2} = \left(\frac{\varepsilon^{'}}{|\varepsilon^*|^2}\Delta\sigma\right)^2 + \left(\frac{\varepsilon^{''}}{|\varepsilon^*|^2}\Delta\sigma\right)^2 + \left(\frac{\sigma^{'} - 2\varepsilon^{'}E^{'}}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 + \left(\frac{\sigma^{''} - 2\varepsilon^{''}E^{'}}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2$$

$$\Delta E^{''2} = \left(\frac{\varepsilon''}{|\varepsilon^*|^2}\Delta\sigma\right)^2 + \left(\frac{\varepsilon'}{|\varepsilon^*|^2}\Delta\sigma\right)^2 + \left(\frac{\sigma^{''} - 2\varepsilon^{'}E''}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 + \left(\frac{-\sigma^{'} - 2\varepsilon^{''}E''}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2$$

which retains complex components. These can be eliminated by gathering terms and applying simplifications originating in the definitions of $E^{'}$, $E^{''}$, $|\varepsilon^*|$ and $|E^*|$:

$$\left(\frac{\varepsilon^{'}}{|\varepsilon^*|^2}\Delta\sigma\right)^2 + \left(\frac{\varepsilon^{''}}{|\varepsilon^*|^2}\Delta\sigma\right)^2 = \left(\frac{\Delta\sigma}{|\varepsilon^*|^2}\right)^2\left(\varepsilon^{'2} + \varepsilon^{''2}\right) = \left(\frac{\Delta\sigma}{|\varepsilon^*|}\right)^2$$

$$\left(\frac{\sigma^{'} - 2\varepsilon^{'}E^{'}}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 + \left(\frac{\sigma^{''} - 2\varepsilon^{''}E^{'}}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 \Longrightarrow \left(|E^*|\frac{\Delta\varepsilon}{|\varepsilon^*|}\right)^2$$

$$\left(\frac{\sigma^{''} - 2\varepsilon^{'}E''}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 + \left(\frac{-\sigma^{'} - 2\varepsilon^{''}E''}{|\varepsilon^*|^2}\Delta\varepsilon\right)^2 \Longrightarrow \left(|E^*|\frac{\Delta\varepsilon}{|\varepsilon^*|}\right)^2$$

Thus, we obtain:

$$\Delta E^{'2} = \Delta E^{''2} = \left(\frac{\Delta\sigma}{|\varepsilon^*|}\right)^2 + \left(|E^*|\frac{\Delta\varepsilon}{|\varepsilon^*|}\right)^2$$

which only contains the magnitude of any complex function and leads directly to the results in Figure 3.