Electronic Supplementary information for *Fibrotaxis*: gradient-free, spontaneous and controllable droplet motion on soft solids

S. R. Bhopalam,[1] J. Bueno,[2] and H. Gomez[1,*]

[1] *School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA.*
[2] *Midge Medical GmbH, Colditzstraße 34/36, 16A, Berlin 12099, Germany.*

(Dated: August 27, 2024)

DROPLET MOTION ON ORTHOTROPIC MATERIALS

We demonstrate droplet motion via fibrotaxis on an anisotropic solid with two families of orthogonal fibers. This specific type of solid represents an orthotropic material [1], where the material properties differ along the directions that are orthogonal to the two families of fibers. In an orthotropic solid, the two families of fibers in the undeformed configuration are oriented along the direction defined by the unit vectors $\boldsymbol{d}$ and $\boldsymbol{d}_\perp$, where $\boldsymbol{d} \cdot \boldsymbol{d}_\perp = 0$. Here, $\boldsymbol{d} = \cos\beta\ \boldsymbol{e}_1 + \sin\beta\ \boldsymbol{e}_2$ where $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the unit vectors in the $x-$ and $y-$directions and β is the angle of the first family of fibers with respect to $\boldsymbol{e}_1$. Similarly, we define $\boldsymbol{d}_\perp = \cos\beta_\perp\ \boldsymbol{e}_1 + \sin\beta_\perp\ \boldsymbol{e}_2$, where $\beta_\perp$ is the angle of the second family of fibers with respect to $\boldsymbol{e}_1$. We define the orientation tensor $\boldsymbol{A}_\perp = \boldsymbol{d}_\perp \otimes \boldsymbol{d}_\perp$ for the second family of fibers accordingly. The anisotropic strain energy density contribution in Eq. (3) of our model for an orthotropic solid is given by

$$W_{\text{aniso}} = \frac{k_1}{k_2} \exp\left[k_2 \left(H-1\right)^2 - 1\right] + \frac{k_1^\perp}{k_2^\perp} \exp\left[k_2^\perp \left(H_\perp - 1\right)^2 - 1\right], \tag{1}$$

where $H_\perp = \text{tr}(\boldsymbol{C}\boldsymbol{A}_\perp)$ is an invariant associated with the second family of fibers, $k_1^\perp$ is a material parameter with dimensions of stress that defines the anisotropy strength of the second family of fibers and $k_2^\perp$ is a dimensionless parameter. We assume that both the families of fibers do not contribute to the material's mechanical response under compression. In our solid model, we assume a continuous and homogeneous distribution of both families of infinite fibers in the solid.

We conduct a two-dimensional simulation of droplet fibrotaxis on an orthotropic solid with two families of orthogonal fibers. The anisotropy strength of the first family of fibers, i.e., k_1 (oriented along β in Fig. 1A) is ten times larger than that of the second family of fibers $k_1^\perp$. Initially, the droplet is placed on the left-hand side of the solid surface. Due to the large anisotropy strength of the first family of fibers, the solid response is softer on the left side of the droplet and stiffer on the right side. Consequently, the droplet spontaneously moves toward the right side of the solid (as shown in Fig. 1). We have confirmed from our numerical simulations that if $k_1 = k_1^\perp$, the droplet velocity will be negligible because the solid's response will be the same at both triple points. In summary, our results show that droplets undergo fibrotaxis on orthotropic materials. Droplet motion on orthotropic materials will be observed only if the anisotropy strength of the two families of fibers is different.

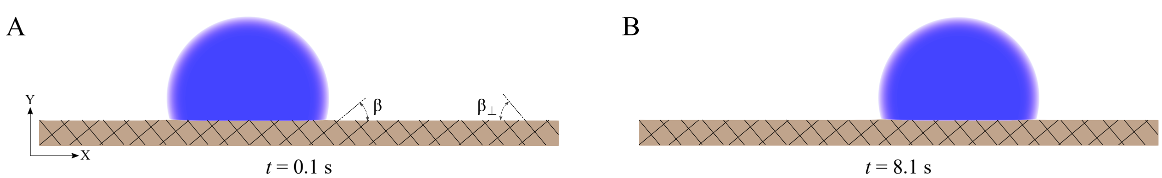

FIG. 1. (A-B) show droplet positions at two times on an orthotropic solid with two families of orthogonal fibers. In (A-B), black solid lines depict the fibers. This depiction does not indicate that we model the fibers discretely; rather, the model assumes that there are infinite fibers continuously distributed. We use the discrete representation of fibers in the figure solely for illustration of their orientation. We use a droplet of radius 160 μm surrounded by air. The droplet velocity measured from the data of droplet position over time is 25 μm/s. The droplet is non-wetting with $\theta = 105^\circ$. The fluids properties are identical to those used in Fig. 3 of the main text. For the solid, we choose $E = 3$ kPa, $\nu = 0.25$, $k_1 = 75$ kPa, $k_2 = 7$, $k_1^\perp = 7.5$ kPa, $k_2^\perp = 7$, $\beta = 60^\circ$, $\beta_\perp = 30^\circ$ and a thickness of 50 μm. We also use $M = 2 \times 10^{-11}$ m^{3}s/kg, $\epsilon = 25$ μm and $\Delta t = 10$ μs.

* Corresponding author; hectorgomez@purdue.edu

[1] G. Holzapfel, *Nonlinear Solid Mechanics: A Continuum Approach for Engineering* (Wiley, 2000).