Supplementary Information (SI) for Soft Matter.

Supplementary Information

Nonlinear Behavior of Stochastic Athermal Fiber Networks with Elastic-Plastic Fibers

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1. Constitutive behavior of the fiber material

Figure S1 shows a typical stress-stretch curve for the fiber material considered, defining the fiber Young's modulus, yield stress and post yield hardening behavior. The curve is described by:

$S^f = E^f \varepsilon^f$, $\varepsilon^f < \varepsilon^f_y$, elastic range

$S^f = S^f_y + E^f_p\left(\varepsilon^f - \varepsilon^f_y\right)$, $\varepsilon^f > \varepsilon^f_y$, plastic range

Hardening in the plastic regime is defined by E^f_p. Unloading takes place on a characteristic line of slope E^f.

A model for the fiber material with constant hardening rate, E^f_p, was preferred to a more realistic model in which the hardening rate decreases with increasing strain because it facilitates the interpretation of the emerging non-linear response of the network. The non-linear response of the network is controlled by the geometric non-linearity associated with large rotations and deformations of the fibers and with the non-linear behavior of the fiber material. Keeping the second type of non-linearity to a minimum while still capturing the yield behavior, makes the effect of the elastic-plastic transition of individual fibers on the overall network response more obvious.

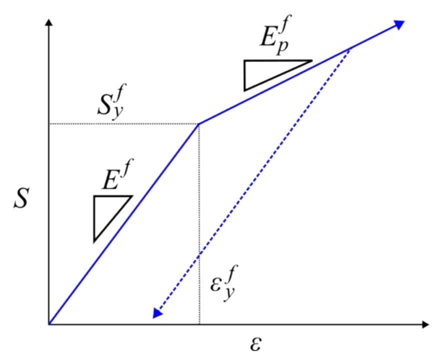

Figure S1. Schematic stress-strain curve for the fiber material showing the slopes of the elastic and plastic regimes.

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2. **Dependence of the network yield stress and strain on the criterion used to identify yielding**

Figure S2 shows the scaling of the network yield stress and yield strain with d/l_c. Two data sets are shown corresponding to the yield point being identified using the 0.2% offset strain (identical to Fig. 2) and based on the strain at which plastically deforming fibers form a percolated path. The percolation-based criterion predicts a network yield strain approximately one order of magnitude larger than the 0.2% offset criterion. The difference between the yield stresses predicted with the two criteria is somewhat smaller than one order of magnitude. However, the scaling of the yield stress and strain with d/l_c is independent of the criterion used. We conclude that the two criteria considered here provide bounds for the yield point, although the 0.2% offset criterion provides a better approximation of actual yield.

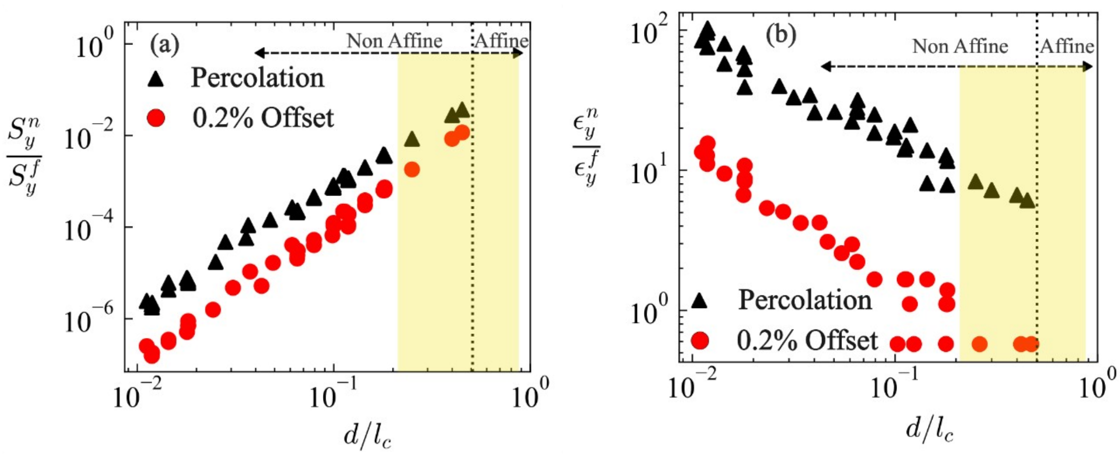

Figure S2. Network yield stress normalized by the yield stress of fibers, S_y^n/S_y^f, vs. d/l_c, and (b) network yield strain normalized by the yield strain of fibers, $\varepsilon_y^n/\varepsilon_y^f$, vs. d/l_c, with the yield point being identified based on the 0.2% offset rule (circles) and based on percolation of plastic paths (triangles).

3. **Percolation of plastically deforming fibers**

As the applied stress increases, an increasing fraction of fibers deform plastically. Percolation of plastic fibers takes place at the points marked by the open triangles in Fig. 3. Figure S3 shows images of a network with $w = -3.54$, $\bar{E}_p^f = 0.07$, $\bar{S}_y^f = 0.005$ before and after percolation, at $\lambda = 1.05$ and $\lambda = 1.15$, respectively. Plastically deforming fibers are shown in color, while elastically deforming fibers are shown in gray.

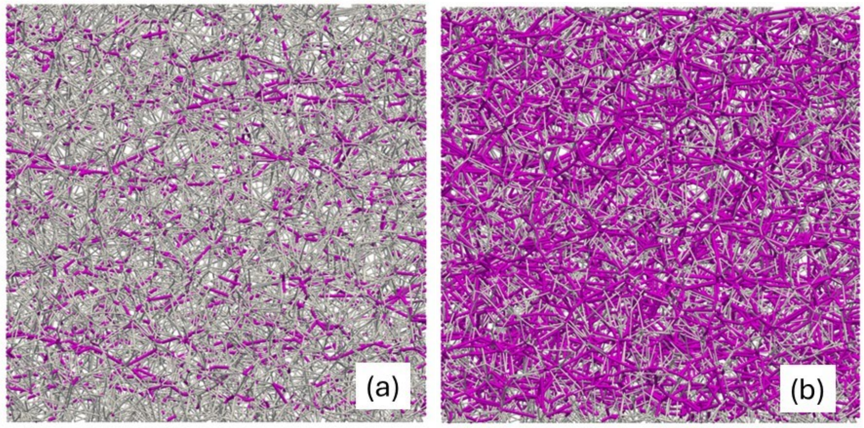

Figure S3. Images of a network with $w = -3.54$, $\bar{E}_p^f = 0.07$, $\bar{S}_y^f = 0.005$ (a) before ($\lambda = 1.05$) and (b) after percolation ($\lambda = 1.15$). The network is mapped back to the undeformed configuration. Plastically deforming fibers are shown in magenta, while elastic fibers are shown in gray. Percolation of plastically deforming fibers takes place at $\lambda = 1.07$.