Supplementary Information

S1] Estimation approach for the total number of photons.

The total number of photons was calculated, considering the photon energy, wavelength, and irradiation time

Photon energy (J)= hc / λ (Eq. S1)

where h is the Planck constant, c is the speed of light, and λ is the photon wavelength.

Photons per second = photon energy (J) / lamp power (J s^{-1}) (Eq. S2)

Total number of photons = photons per second (photons s^{-1})· reaction time (s^{-1}) (Eq. S3)

Table S1. Specifications of the filter and air purifier (AP) codes for the AP platform.

Order	AP code	Filter code	Filter type
1	AP (CNT-n)	HC-CNT-n	HC filter fabricated using CNT-n photocatalyst.
2	AP (P25)	HC-TiO_2	HC filter fabricated using TiO_2 in the lab
3	AP (CN)	HC-g-C_3N_4	HC filter fabricated using g-C_3N_4 photocatalyst
4	AP (O)	HC-O	HC filter installed in commercial AP
5	AP (B)	HC-B	Blank honeycomb filter

Table S2. Natural decay rate of FA vapor at varying concentrations.

Order	Initial concentration (ppm)	R^2	Natural decay rate ($L\ min^{-1}$)
1	0.5	0.948	0.009
2	1	0.983	0.011
3	2	0.997	0.011
4	5	0.987	0.015

Table S3. Surface properties of the tested CNT-n, TiO_2, and g-C_3N_4 photocatalysts.

Order	Catalyst	S_{BET} (m^2 g^{-1})	Pore volume (cm^3 g^{-1})	Average pore diameter (nm)
1	CNT-1	41.6	0.0505	4.49
2	CNT-0.5	42.9	0.0572	4.51
3	CNT-0.1	45.3	0.0581	4.65
4	CNT-0.02	46.1	0.0612	4.78
5	TiO_2	50.8	0.0678	4.44
6	g-C_3N_4	9.57	0.0124	4.37

Table S4. XPS data for the CNT-0.02, g-C_3N_4, and TiO_2 catalysts.

Sample	Peak	Binding energy (eV)	Assignment	Element (atomic %)
CNT-0.02	C 1s	285	CC/C=C	13.3
		286.5	CN	
		288.5	$C(N)_3$	
	N 1s	399.2	CN=C	4.47
		401.1	$N(C)_3$	
	Ti 2p	458.4	Ti $2p_{3/2}$	56.2
		464.2	Ti $2p_{1/2}$	
	O 1s	529.5	Ti-O	26.1
		531.7	Ti-OH	
g-C_3N_4	C 1s	285	CC/C=C	44.5
		286.5	CN	
		288.5	$C(N)_3$	
	N 1s	398.9	CN=C	49.2
		400.8	$N(C)_3$	
TiO_2	Ti 2p	458.5	Ti $2p_{3/2}$	16.4
		464.3	Ti $2p_{1/2}$	
	O 1s	529.6	Ti-O	41.7
		531.8	Ti-OH	

Table S5. Intermediates identified through an *in situ* DRIFTS analysis of CNT-0.02 during FA removal in dark and light conditions.

Order	Intermediate	Wavelength (cm^{-1})	Assignment	Note
1	Dioxymethylene (DOM)	1064, 1116, 1158, 1174, and 1253	ν(CO)	1. An increase in the DOM peak during the adsorption process indicates the surface consumption of bonded OH groups, allowing the adsorption of DOM species onto the photocatalytic surface. 2. Under UV exposure, peaks of DOM species and formate ions decrease and increase, respectively, indicating the conversion of DOM species into formate ions due to oxidation. 3. The introduction of humidity leads to an increase in OH peaks, forming hydroxyl groups, followed by a decrease in DOM and formate species.
		1417	$\delta(CH_2)$	
		2762, 2861, 2911, and 2960	$\nu(CH_2)$	
2	Formate ($HCOO^-$)	1360	$\nu s(COO^-)$	
		1572	$\nu as(COO^-)$	

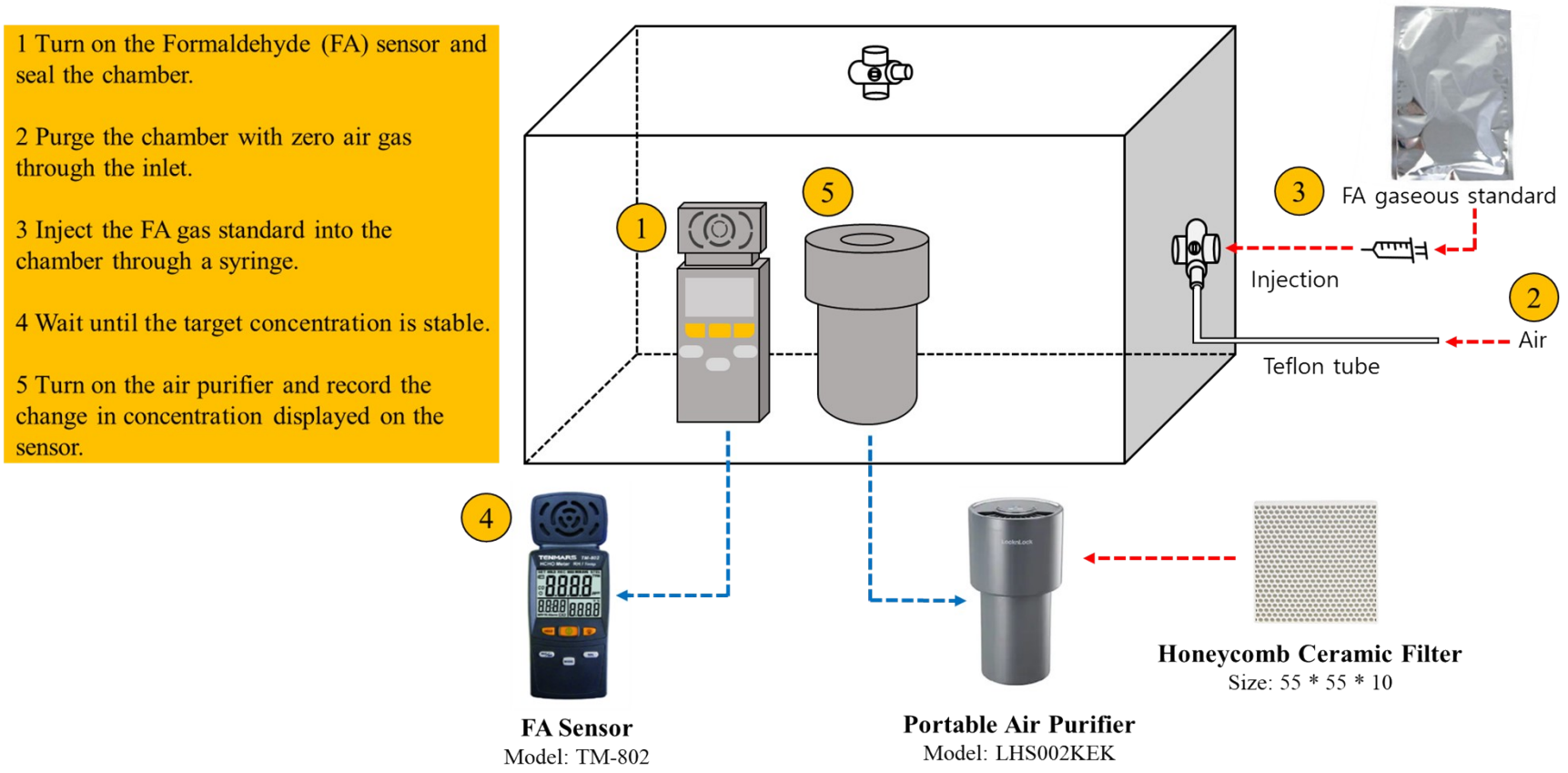


Figure S1. Schematic diagram of a photocatalytic oxidation system for FA removal.

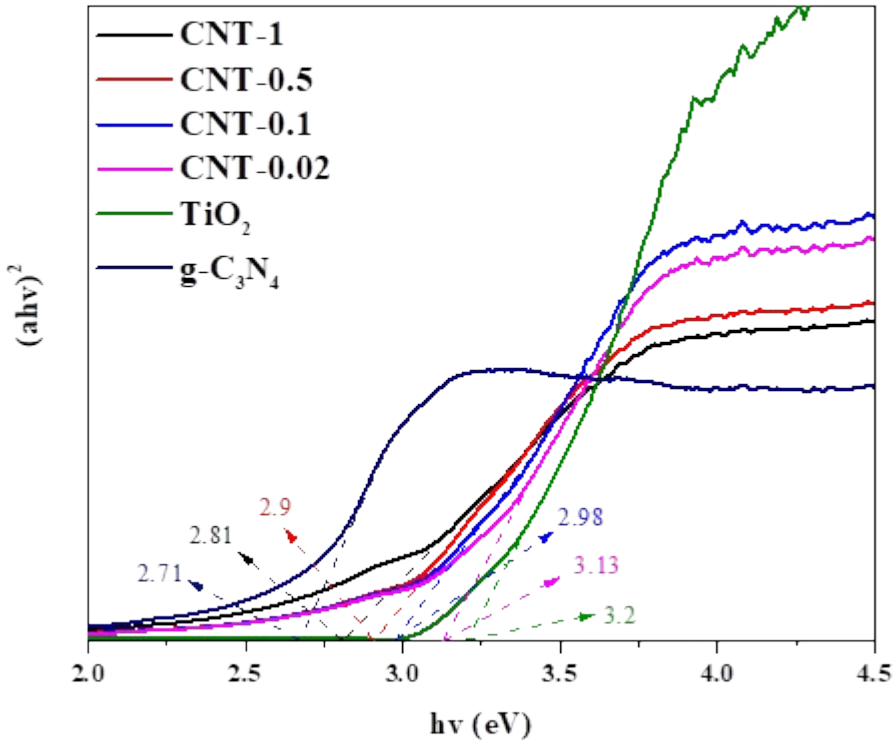


Figure S2. Band gap energy profiles for the as-prepared CNT-n composites (relative to TiO_2 and g-C_3N_4).

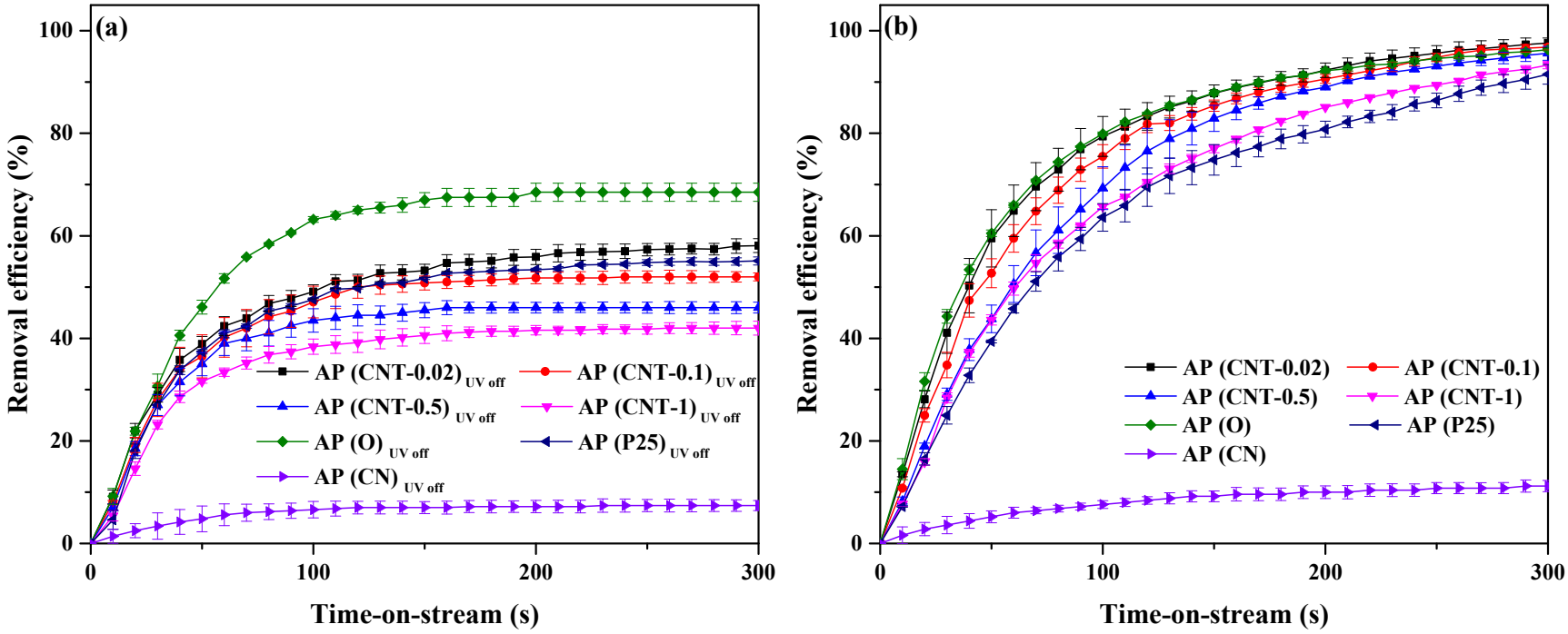


Figure S3. The removal efficiency of 5 ppm FA by AP (CNT-n) filters (relative to the reference AP (O), AP (P25), and AP (CN) filters) in a dry atmosphere (0% RH) at 160 mL min^{-1} AP circulation rate: (a) adsorption in dark conditions and (b) photocatalysis under 42.1 mW cm^{-2} UV-A irradiation.

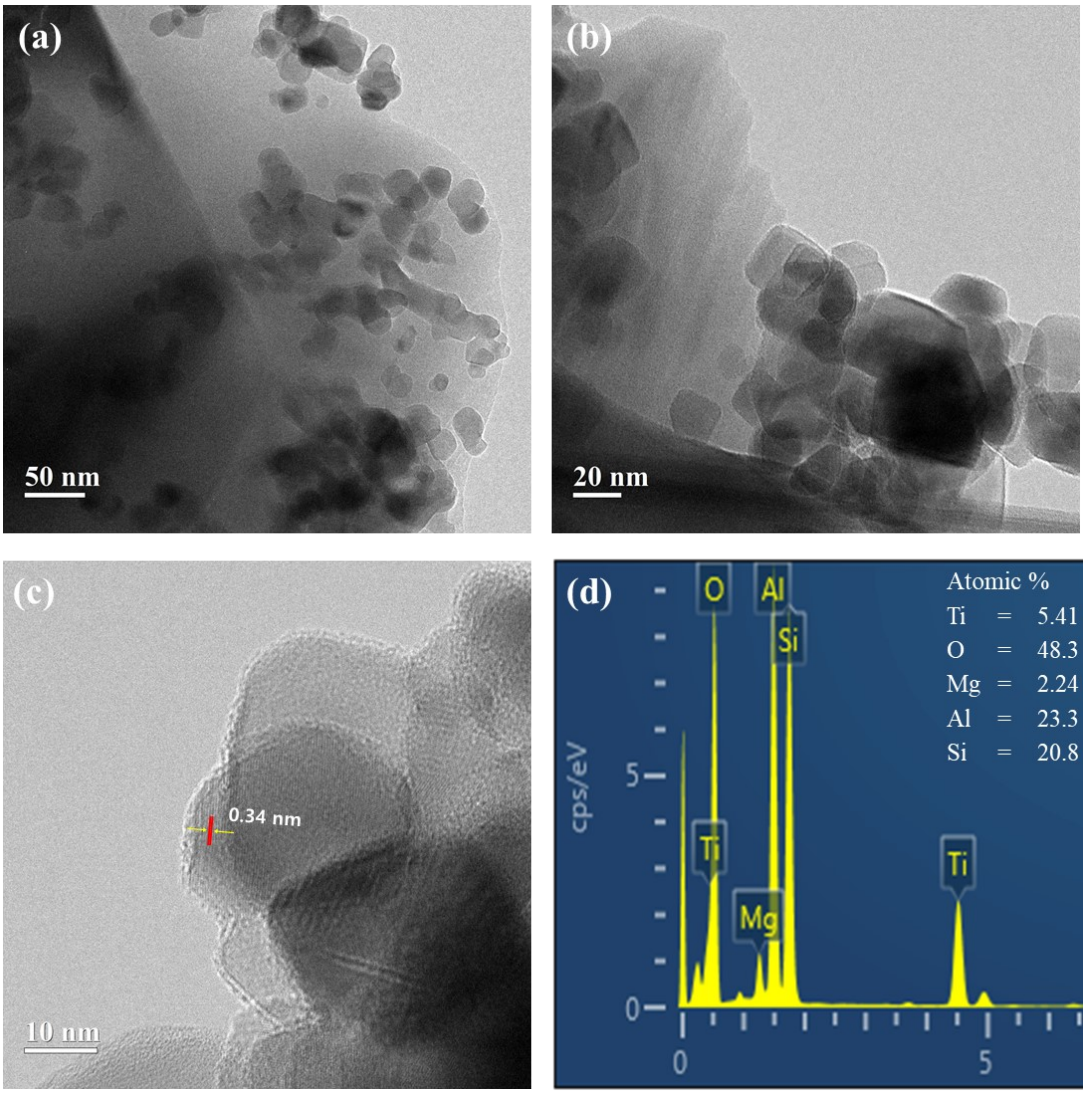
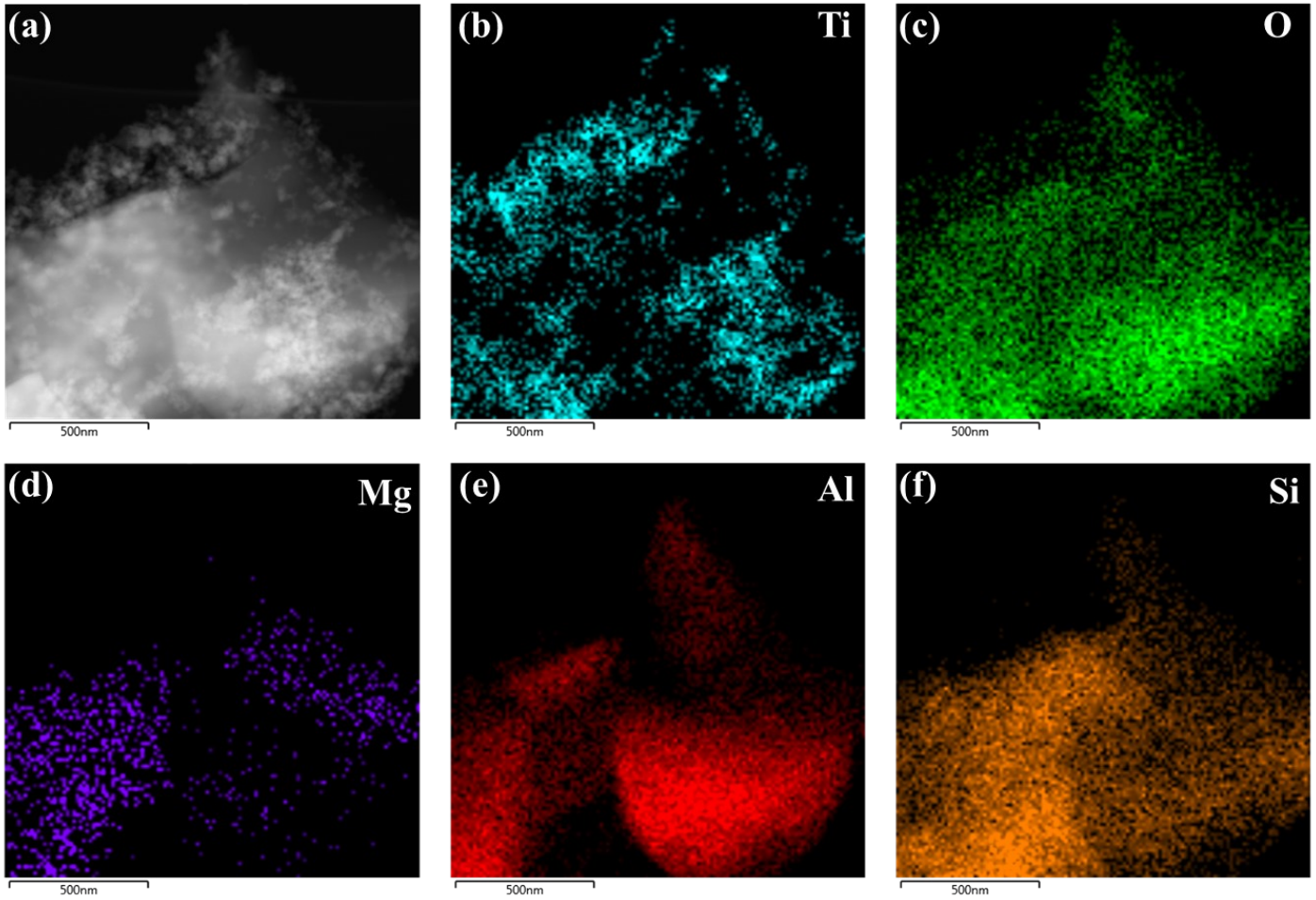


Figure S4. Morphological characterization of the HC-O material: (a–c) TEM images and (d) EDS analysis.



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41 **Figure S5.** EDS images of HC-O (a) Selected region and (b) Ti, (c) O, (d) Mg, (e) Al, and (f) Si

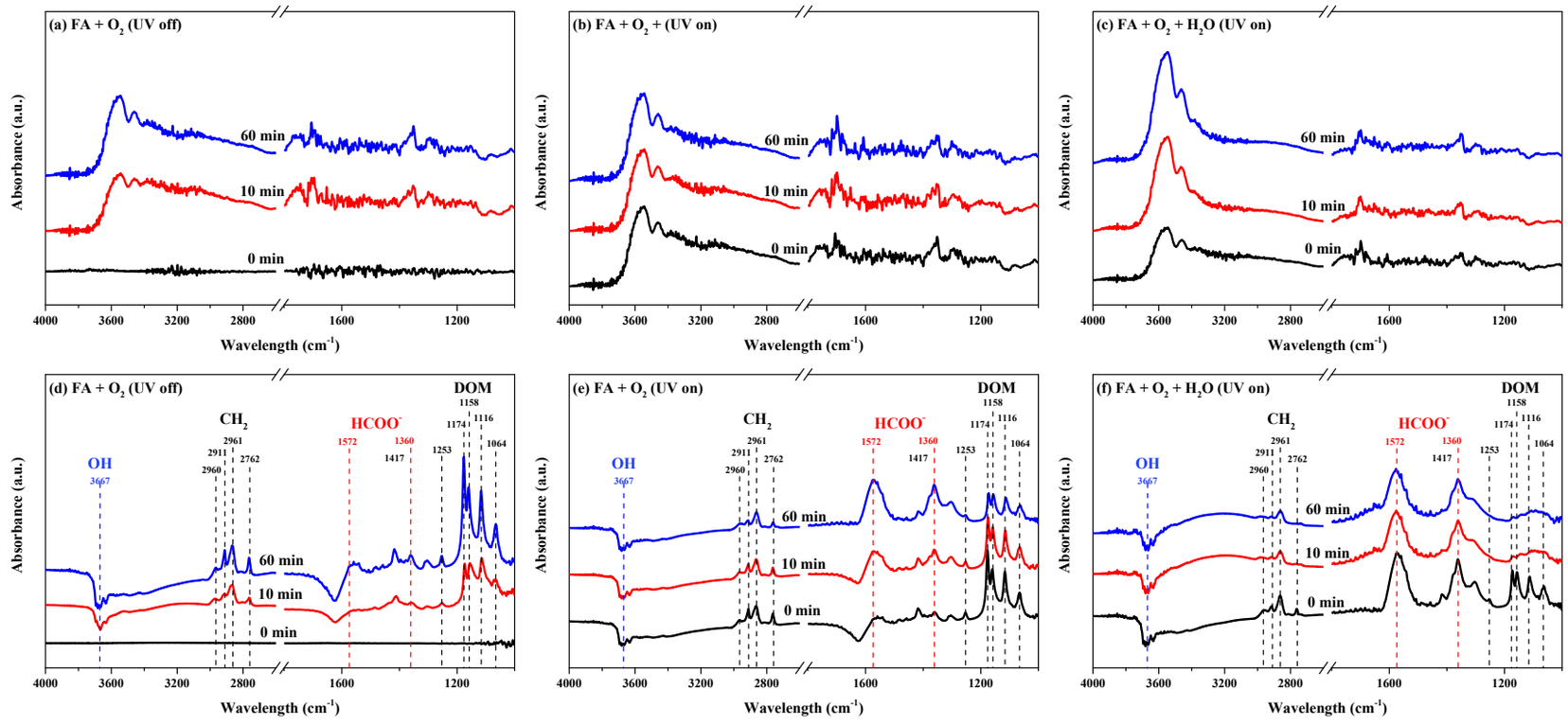

Figure S6. *In situ* DRIFTS spectra of the tested photocatalysts against FA removal in dark and light conditions: (a) g-C_3N_4 (FA + air + UV off), (b) g-C_3N_4 (FA + air + UV on), (c) g-C_3N_4 (FA + air + H_2O + UV on), (d) TiO_2 (FA + air + UV off), (e) TiO_2 (FA + air + UV on), and (f) TiO_2 (FA + air + H_2O + UV on).