Supplemental Information for "Tunable light-induced topological edge states in strain engineering bismuthene monolayers"

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Fig. S1. The side and top views for (a) Bi(110) and (b) Bi(111) monolayers.

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Fig. S2. Bi(100) monolayer with lattice parameters from Ref. [1], (a) the edge states along [100] direction for both edge terminations; (c) Wannier Charge Centers (WCCs) spectrums that evolve along [010] and [100] directions. For lattice parameters from Ref. [2], (b) the edge states along [100] direction for both edge terminations; (d) Wannier Charge Centers (WCCs) spectrums that evolve along [010] and [100] directions. The red circles indicate the gap in the WCC spectrums.



Fig. S3. Bi(110) monolayer with lattice parameters from Ref. [3], (a) the electronic band structures; (b) the WCCs spectrum along [010] direction; (c) and (d) the edge states for both right and left edges along [010] and [100] directions, respectively.



Fig. S4. Bi(110) monolayer with lattice parameters and 5% tensile strain from Ref. [3], (a) shows the Floquet band structures (red lines) with light intensity, and the gray lines in (a) indicate the undriven electronic band structures. (b) shows the double pairs of helical edge states in the undriven Bi(110) monolayer on the right edge along along [010] direction. (c) When $eA/\hbar = 0.5\text{\AA}^{-1}$, the light-bismuthene coupled system behaves only one pair of helical edge states along [010] direction.



Fig. S5. With 4% tensile strain on Bi(111) monolayer, (a) shows the Floquet band structures (red lines) with light intensity $eA/\hbar = 0.25 \text{\AA}^{-1}$, 0.35\AA^{-1} and 0.5\AA^{-1} , respectively; the gray lines in (a) indicate the undriven electronic band structures. (b) the Floquet engineering edge states with light intensity $eA/\hbar = 0 \text{\AA}^{-1}$, 0.25\AA^{-1} along [010] direction. (c) the corresponding WCCs spectrum with light intensity $eA/\hbar = 0 \text{\AA}^{-1}$, 0.25\AA^{-1} , 0.25\AA^{-1} and 0.5\AA^{-1} along [010] direction.



Fig. S6. The comparison between band structures from DFT calculations and Wannier functions indicates the projections of p orbitals in Bi atoms could fully describe the Bi(110) and Bi(111) monolayers.



Fig. S7. The light-induced band gaps in (a) Bi (110) monolayer with lattice parameters from Ref. [3]; (b) Bi(111) monolayer with fully relaxed lattice parameters (a = 4.33Å) and (c) Bi(111) monolayer with 4% compressive strain. Due to the topological phase transitions relates with the opening and closing of local band gaps, so we define the light-induced band gaps in Bi(111) monolayer as the local gap around Γ point.