

**Supplementary Information of**  
**Advanced Thermal/Environmental Barrier Coatings of High-Entropy Rare Earth Disilicates Tuned**  
**by Strong Anharmonicity of Eu<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>**

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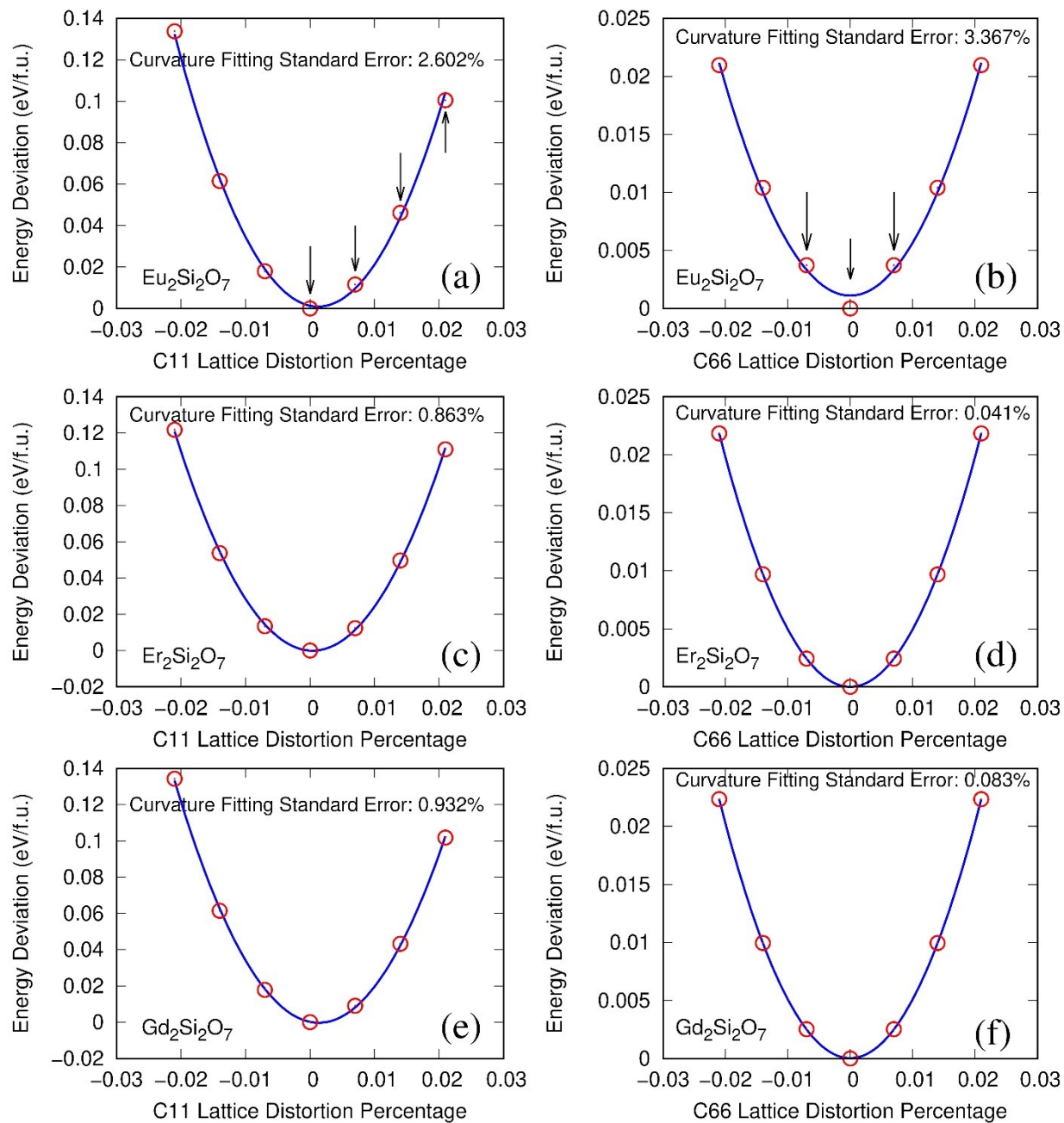
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The C<sub>11</sub> and C<sub>66</sub> type lattice distortions denoted as  $LD_{C11}$  and  $LD_{C66}$  alter the lattice parameters according to the following matrices:

$$LD_{C11} = \begin{pmatrix} 1 + \delta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; LD_{C66} = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where  $\delta$  values are set to  $\pm 0.007$ ,  $\pm 0.014$ , and  $\pm 0.021$  in percentage. The resulting energy deviation ( $\Delta E = E(\delta) - E(\delta=0)$ ) versus  $\delta$  for Eu<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, Er<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, and Gd<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> are shown in Figure S1.



**Fig. S1** The potential energy deviation relative to the fully relaxed equilibrium states as the lattice undergoes a series of C<sub>11</sub> and C<sub>66</sub> type lattice distortions for (a-b) Eu<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, (c-d) Er<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>, and (e-f) Gd<sub>2</sub>Si<sub>2</sub>O<sub>7</sub>. The much larger energy deviation points from the fitted parabolic lines are highlighted in (a) and (b) by arrows. The much larger curvature fitting standard errors of Eu<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> than Er<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> and Gd<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> suggest that Eu<sub>2</sub>Si<sub>2</sub>O<sub>7</sub> lattice exhibits much stronger lattice anharmonicity.