

Supplementary for “Prospects for photon-phonon dressing and crystal-field non-Hermitian alignment of Eu³⁺: BiPO₄”

Alignment theoretical model

The cascaded double dressing formula of second order FL and third order SFWM for CF levels ($M_j = 0, +1$) is written below.

$$\rho_{FL}^{(+2)} = |G_i^+|^2 / (\Gamma_{20}^+ + i\Delta_1^+) (\Gamma_{22}^+ + |G_i^+|^2 / (\Gamma_{20}^+ + i\Delta_1^+) + |G_{p1}^+|^2 / (\Gamma_{10}^+ + i\Delta_1^+ - i\Delta_{p1}^+)) \quad (S1)$$

$$\rho_{FL}^{(02)} = |G_i^0|^2 / (\Gamma_{21}^0 + i\Delta_1^0) (\Gamma_{22}^0 + |G_i^0|^2 / (\Gamma_{21}^0 + i\Delta_1^0) + |G_{p1}^0|^2 / (\Gamma_{01}^0 + i\Delta_1^0 - i\Delta_{p1}^0) + |G_{p2}^0|^2 / (\Gamma_{31}^0 + i\Delta_1^0 + i\Delta_{p2}^0)) \quad (S2)$$

$$\rho_{S/AS}^{(+3)} = -iG_{AS}^+ G_i^+ G_i^+ / (\Gamma_{20}^+ + i\Delta_1^+) (\Gamma_{00}^+ + i\Delta_1^+ + |G_i^+|^2 / (\Gamma_{20}^+ + i\delta_{S/AS}) + |G_{p1}^+|^2 / (\Gamma_{10}^+ + i\delta_{S/AS} - i\Delta_{p1}^+)) (\Gamma_{20}^+ + i\delta_{S/AS} + i\Delta_1^+) \quad (S3)$$

$$\rho_{S/AS}^{(03)} = -iG_{AS}^0 G_i^0 G_i^0 / (\Gamma_{21}^0 + i\Delta_1^0) (\Gamma_{11}^0 + i\Delta_1^0 + |G_i^0|^2 / (\Gamma_{21}^0 + i\delta_{S/AS}) + |G_{p1}^0|^2 / (\Gamma_{01}^0 + i\delta_{S/AS} - i\Delta_{p1}^0)) (\Gamma_{20}^0 + i\delta_{S/AS} + i\Delta_1^0) \quad (S4)$$