# Supporting Information

**Shear-Structured Piezoelectric Accelerometers based on KNN Lead-Free Ceramics for Vibration Monitoring**

*Yi Ding[1], Yu Wang[1], Wenbin Liu[1], Yongqi Pan[1], Ping Yang[3], Dechao Meng[3]*, Ting Zheng[1]* and Jiagang Wu[1,2]**

1. College of Materials Science and Engineering, Sichuan University; Chengdu, Sichuan, 610064, China.

2. College of Physics, Sichuan University; Chengdu, Sichuan, 610064, China.

3. Microsystem and Terahertz Research Center & Institute of Electronic Engineering, China Academy of Engineering Physics, Chengdu 610200, P. R. China

***Corresponding Author: E-mail**: wujiagang0208@163.com (J. G. Wu); zhengtingscu@126.com (T. Zheng); mdechao@163.com (D. C. Meng)

**Formulas for calculating other parameters:**

The mechanical quality factor $Q_m$ can be calculated using the following formula[1, 2]:

$$Q_m = \frac{f_p^2}{2\pi |Z_m| (C_0 + C_1) f_s (f_p^2 - f_s^2)} \quad (1)$$

The corresponding mode is radial expansion mode, where $f_s$=$f_r$, $f_p$=$f_a$, $|Z_m|$ is the impedance corresponding to the resonant frequency, and ($C_0$+$C_1$)is the low-frequency capacitance.

The free dielectric impermeability constants $\beta_{ii}^T$ can be calculated using the following formula[1]:

$$\beta_{ii}^T = 1/\varepsilon_{ii}^T \quad (2)$$

The piezoelectric voltage coefficient $g_{ij}$ can be calculated using the following formula[1-3]:

$$g_{ij} = d_{ij}\beta_{ii}^T \quad (3)$$

Other elastic compliance coefficients and stiffness compliance coefficients can be calculated using the following formula[1-3]:

$$s_{11}^D = s_{11}^E (1 - k_{31}^2) \quad (4)$$

$$c_{44}^E = c_{44}^D (1 - k_{15}^2) \quad (5)$$

$$s_{44}^{E} = 1/c_{44}^{E} \tag{6}$$

$$s_{44}^{D} = 1/c_{44}^{D} \tag{7}$$

$$s_{12}^{E} =- \sigma^{E} \cdot s_{11}^{E} \tag{8}$$

$$s_{12}^{D} = s_{12}^{E} - k_{31}^{2} \cdot s_{11}^{E} \tag{9}$$

$$s_{66}^{E} = s_{66}^{D} = \frac{1}{c_{66}^{E}} = \frac{1}{c_{66}^{D}} = 2\left(s_{11}^{E} - s_{12}^{E}\right) \tag{10}$$

$$s_{13}^{D} =- \left\{\frac{s_{11}^{D} + s_{12}^{D}}{2} \cdot \left(s_{33}^{D} - \frac{1}{c_{33}^{D}}\right)\right\}^{0.5} \tag{11}$$

$$s_{13}^{E} = s_{13}^{D} + d_{33}g_{31} = s_{13}^{D} + k_{31} \cdot k_{33}\left(s_{33}^{E} \cdot s_{11}^{E}\right)^{0.5} \tag{12}$$

$$c_{11}^{E} = \frac{s_{11}^{E}s_{33}^{E} - \left(s_{13}^{E}\right)^{2}}{\left(s_{11}^{E} - s_{12}^{E}\right)\left[s_{33}^{E}\left(s_{11}^{E} + s_{12}^{E}\right) - 2\left(s_{13}^{E}\right)^{2}\right]} \tag{13}$$

$$c_{12}^{E} = \frac{- s_{12}^{E}s_{33}^{E} + \left(s_{13}^{E}\right)^{2}}{\left(s_{11}^{E} - s_{12}^{E}\right)\left[s_{33}^{E}\left(s_{11}^{E} + s_{12}^{E}\right) - 2\left(s_{13}^{E}\right)^{2}\right]} \tag{14}$$

$$c_{13}^{E} = \frac{- s_{13}^{E}}{s_{33}^{E}\left(s_{11}^{E} + s_{12}^{E}\right) - 2\left(s_{13}^{E}\right)^{2}} \tag{15}$$

$$c_{11}^{D} = c_{11}^{E} + h_{31}e_{31} \tag{16}$$

$$c_{12}^{D} = c_{12}^{E} + h_{31}e_{31} \tag{17}$$

$$c_{13}^{D} = c_{13}^{E} + h_{31}e_{33} \tag{18}$$

The piezoelectric stress coefficients $e_{ij}$, and piezoelectric stiffness coefficients $h_{ij}$ can be calculated using the following formula[1-3]:

$$e_{31} = d_{31}\left(c_{11}^{E} + c_{12}^{E}\right) + d_{33}c_{13}^{E} \tag{19}$$

$$e_{33} = 2d_{31}c_{13}^{E} + d_{33}c_{33}^{E} \tag{20}$$

$$e_{15} = d_{15}c_{44}^{E} \tag{21}$$

$$h_{ij} = e_{ij}\beta_{ii}^{S} \tag{22}$$

The frequency constant is primarily used to describe the relationship between the resonant frequency and size. The following formulas represent the frequency constants for different vibration modes[1, 2]:

1) Lateral displacement vibration mode

$$N_1 = f_r \cdot l \tag{23}$$

2) Longitudinal vibration mode of a thin rod

$$N_3 = f_r \cdot l \tag{24}$$

3) Shear vibration mode

$$N_5 = f_r \cdot t \tag{25}$$

4) Planar radial vibration mode

$$N_p = f_r \cdot d \tag{26}$$

5) Thickness vibration mode

$$N_t = f_r \cdot t \tag{27}$$

**Supplementary images:**

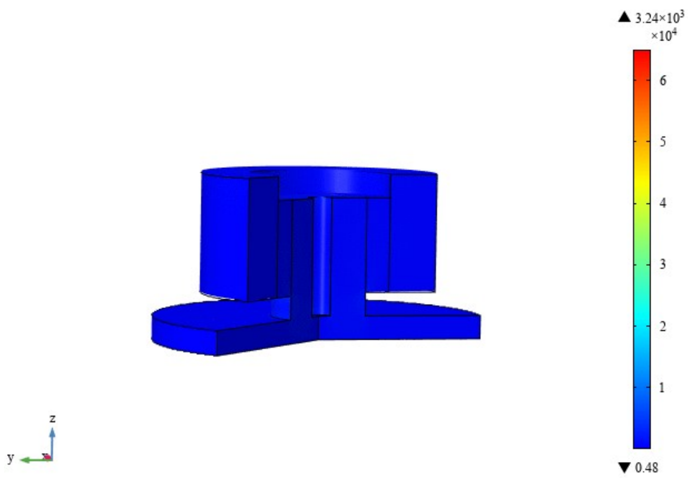


Figure S1. 3D simulated stress distribution map of finite element simulation under different accelerations of 1g-20g.

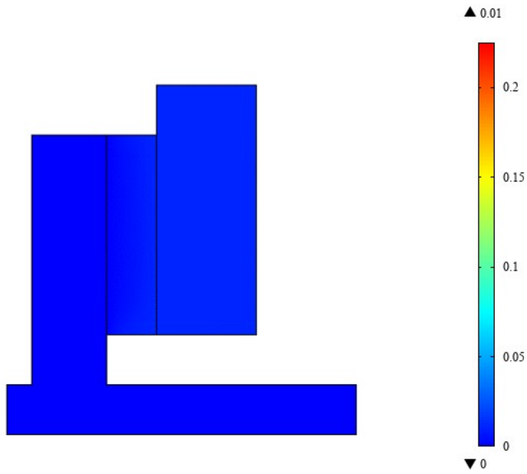


Figure S2. Charge distribution map of finite element simulation under different accelerations of 1g-20g.


Figure S3. Physical picture of piezoelectric accelerometer testing system.

**References**

[1] ZHANG S, ALBERTA E F, EITEL R E, et al. Elastic, piezoelectric, and dielectric characterization of modified BiScO3-PbTiO3 ceramics [J]. ieee transactions on ultrasonics, ferroelectrics, and frequency control, 2005, 52(11): 2131-2139.
[2] BOARD I S. IEEE Standard on Piezoelectricity [J]. 1996.
[3] QIAO L, LI G, TAO H, et al. Full characterization for material constants of a promising KNN-based lead-free piezoelectric ceramic [J]. Ceramics International, 2020, 46(5): 5641-5644.