## SUPPLEMENTARY INFORMATION FOR

## Interface energies of $Ga_2O_3$ phases with the sapphire substrate and the phase-locked epitaxy of metastable structures explained

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phase	a[Å]	b[Å]	c[Å]
$\kappa - Ga_2O_3$	5.061	8.686	9.292
$\beta - Ga_2O_3$	14.763	3.048	5.809
$\alpha - Ga_2O_3$	5.001	5.001	13.448
$\alpha - Al_2O_3$	4.775	4.775	13.015

Table S1: Lattice parameters of optimized bulk cells, oriented according to the substrate. The slabs of the film were built in order to align the  $\beta$  [102] along the  $\alpha$  [100], the  $\beta$  [010] along the  $\alpha$  [120], the  $\kappa$  [100] along the  $\alpha$  [100] and the  $\kappa$  [010] along the  $\alpha$  [120].

		$m_x / m_y$ misfit strain			
phase	x/y axis	vs $\alpha - Al_2O_3$	vs $\alpha - Ga_2O_3$	vs $\beta - Ga_2O_3$	
$\kappa - Ga_2O_3$	[100]/[010]	4.8 % / 5.7 %	0.3 % / -1.2 %	2.8~% / -5.3 $%$	
$\beta - Ga_2O_3$	[102]/[010]	9.5 % / 3.0 %	5.3 % / -1.6 %	-	
$\alpha - Ga_2O_3$	[100]/[120]	4.5 % / 4.5 %	-	-	
$\alpha - Al_2O_3$	[100]/[120]	-	-	-	
	·	epitaxial relationship: $n_x \times n_y$ (film): $n_x \times n_y$ (substrate)			
phase	in-plane cell	vs $\alpha - Al_2O_3$	vs $\alpha - Ga_2O_3$	vs $\beta - Ga_2O_3$	
$\kappa - Ga_2O_3$	conventional-rectangular	1×1:1×1	1×1:1×1	1×1:1×1	
$\beta - Ga_2O_3$	conventional-rectangular	$1 \times 3:3 \times 1$	$1 \times 3:3 \times 1$	$1 \times 3:3 \times 1$	
$\alpha - Ga_2O_3$	primitive-hexagonal	1×1:1×1	1×1:1×1	1×1:1×1	

Table S2: Misfit strain with the substrate, calculated as:  $m_i = (a_i^{film} - a_i^{substr})/a_i^{film}$ .  $a_i$  is the lattice parameter of the film/substrate along the given direction.

phase	$\Delta \mu_{\varepsilon} \ [meV/f.u.]$	$\gamma_{\rm epi} \ [{\rm meV/f.u.}]$	$v [Å^3/f.u.]$	h [Å]				
on $\alpha - Al_2O_3$ substrate								
$\alpha$	309	88	45.8	2.32				
β	540	57	48.6	4.93				
$\kappa$	314	86	48.1	4.88				
on $\alpha - \text{Ga}_2\text{O}_3^{\varepsilon}$ substrate								
$\alpha$	309	88	45.8	2.32				
$\beta$	540	57	48.6	4.93				
$\kappa$	314	86	48.1	4.88				
on $\alpha - Ga_2O_3$ substrate								
$\alpha$	0	70	48.5	2.24				
β	150	53	51.3	4.74				
$\kappa$	10	53	50.8	4.69				
$lpha - \mathrm{Al}_2\mathrm{O}_3: \ \gamma_{epi} = 113 meV/\mathrm{\AA}^2$								

Table S3: Elastic energy, surface energy of the epilayer, atomic volume of the cell and thickness of the layer for the different phases, given  $\alpha - Al_2O_3$  substrate,  $\alpha - Ga_2O_3$  strained interlayer and  $Ga_2O_3$  fully relaxed interlayer.



Figure S1: Front (a) and side (b) view of the  $\beta - Ga_2O_3/\alpha - Al_2O_3$  interface nr. 2. The black line marks the plane of O atoms shared by both film and substrate.



Figure S2: Front (a) and side (b) view of the  $\kappa - Ga_2O_3/\alpha - Al_2O_3$  interface nr. 2. The black line marks the plane of O atoms shared by both film and substrate.



Figure S3: Front (a) and side (b) view of the  $\beta - Ga_2O_3/\alpha - Ga_2O_3$  interface. The  $\beta - Ga_2O_3$  interlayer is fully strained on  $\alpha - Al_2O_3$  substrate. The black line marks the plane of O atoms shared by both film and substrate.



Figure S4: Front (a) and side (b) view of the  $\beta - Ga_2O_3/\alpha - Ga_2O_3$  interface. The  $\beta - Ga_2O_3$  interlayer is fully relaxed. The black line marks the plane of O atoms shared by both film and substrate.



Figure S5: Front (a) and side (b) view of the  $\kappa - Ga_2O_3/\alpha - Al_2O_3$  interface. The  $\alpha - Ga_2O_3$  interlayer is fully strained on  $\alpha - Al_2O_3$  substrate. The black line marks the plane of O atoms shared by both film and substrate.



Figure S6: Front (a) and side (b) view of the  $\kappa - Ga_2O_3/\alpha - Al_2O_3$  interface. The  $\alpha - Ga_2O_3$  interlayer is fully relaxed. The black line marks the plane of O atoms shared by both film and substrate.



Figure S7: Front (a) and side (b) view of the  $\kappa - Ga_2O_3/\beta - Ga_2O_3$  interface. The  $\beta - Ga_2O_3$  interlayer is fully relaxed. The black line marks the plane of O atoms shared by both film and substrate.

The files of the optimized structures reported in the main manuscript and in the Supplementary Information are collected in the archive supplementary\_information\_geometries.zip