Supplementary Information

Reduced lattice thermal conductivity and strong four-phonon scattering in h-B₁₂ assembled by boron clusters on a honeycomb lattice

Ting Zhang^a, Yu-Run Yang^a, Xu Liu^a, Jing Wang^{*a}, Zhao Liu^{*ab}, and Ying Liu^{ac}

^a Department of Physics and Hebei Advanced Thin Film Laboratory, Hebei Normal University, Shijiazhuang, 050024, China

^b Beijing Computational Science Research Center, Beijing, 100193, China

^c National Key Laboratory for Materials Simulation and Design, Beijing, 100083, China

* Corresponding author: <u>zliu@hebtu.edu.cn</u> and <u>jwang@hebtu.edu.cn</u>

SI. Convergence calculations of κ_{lat} about q-points.

We can see that the κ_{lat} of 2D *h*-B₁₂ is converged to within 5% for an increase in **q**-points beyond 90 × 90 × 1 when considering only 3ph scattering. Additionally, it is converged to within 5% for an increase in **q**-points beyond 22 × 22 × 1 when considering both 3ph and 4ph scatterings.



Fig. S1 Variation of κ_{lat} with different **q**-points for 2D *h*-B₁₂, considering (a) only 3ph scattering and (b) both 3ph and 4ph scattering processes. The red arrow indicates the selected **q**-points, and the green dashed line represents a fluctuation of $\pm 5\%$ around the converged value.

S2. Basic information on the three structures assembled from icosahedral B₁₂ clusters.

Figure S2(a) displays the optimized 2D honeycomb structure of $h-B_{12}$, the triangular structure of t-B₁₂, and the kagome structure of k-B₁₂ from left to right. All three structures are assembled from the icosahedral B_{12} clusters as the basic building unit. Table S1 lists the space group, lattice constants, atomic layer thickness, and the B-B bonding lengths for adjacent clusters after comprehensive optimization for *h*-B₁₂, t-B₁₂, and k-B₁₂. Figure S2(b) shows the phonon dispersions of the three structures in Fig. S2(a), We found that only $h-B_{12}$ has no imaginary frequencies, proving its stability. Figure S2(c) displays the band structures of three structures in Fig. S2(a) calculated using the PBE0 functional, revealing that the honeycomb $h-B_{12}$ is an indirect bandgap semiconductor ($\Delta = 1.07 \text{ eV}$), while the triangular *t*-B₁₂ and kagome $k-B_{12}$ are metallic. Therefore, the main carriers of thermal transport in $h-B_{12}$ are phonons. Notably, at the K point, there are two Dirac states labeled as D₁ (1.7 eV above the Fermi level) and D₂ (1.4 eV below the Fermi level). Furthermore, the group velocities at D₁ (2.75×10^5 m/s) and D₂ (1.31×10^5 m/s) are only one order of magnitude lower than the Fermi velocity in graphene,¹ suggesting that $h-B_{12}$ is likely to exhibit high electrical conductivity. Additionally, h-B₁₂ exhibits a stronger spin-orbit coupling (SOC) effect compared to graphene and silicon, with a Rashba coefficient of approximately 1.8 eV/Å, which is greater than or at least comparable to that of many well-known strong SOC materials.²⁻⁶ The aforementioned results can also be found in Fig. 2 of Ref. 32 in the manuscript. This work presents the thermal



transport properties of h-B₁₂, using first-principles calculations in conjunction with the Boltzmann Transport Equation (BTE) method.

Fig. S2 (a) Top and side views of 2D h-B₁₂ (left), t-B₁₂ (middle), and k-B₁₂ (right), with d representing the atomic layer thickness, the black rhombic dashed box indicates the unit cell. (b) Phonon dispersions of 2D h-B₁₂ (left), t-B₁₂ (middle), and k-B₁₂ (right). (c) Band structures of 2D h-B₁₂ (left), t-B₁₂ (middle), and k-B₁₂ (right). (c) Band structures of 2D h-B₁₂ (left), t-B₁₂ (middle), and k-B₁₂ (right), referenced to the Fermi energy. In the band structure of h-B₁₂, two Dirac states are marked as D₁ (1.7 eV above the Fermi level) and D₂ (1.4 eV below the Fermi level).

Structure	Space	a (Å) ^{a)}	b (Å) ^{a)}	d (Å) ^{a)}	B-B (Å) ^{a)}
	group				
<i>h</i> -B ₁₂	$P\overline{3}1m$	8.28	8.28	2.45	1.71
<i>t</i> -B ₁₂	$P\overline{3}m1$	4.78	4.78	2.46	1.93
k-B ₁₂	<i>C2/m</i>	9.49	9.20	3.00	1.69

Table S1. The parameters of the three structures in Fig. S2(a).

a) "*a*" and "*b*" represent the lattice constants, "*d*" is the thickness of the atomic layer, and "B-B" refers to the bond length of B-B bonds connecting adjacent clusters.

S3. Three types of bonds in *h*-B₁₂.

The unit cell of h-B₁₂ contains two mirror-symmetric B₁₂ clusters. Each B₁₂ cluster in this structure is connected to three adjacent B₁₂ clusters through two B–B bonds, with a bond length of 1.71 Å, which is in the range of B-B σ bonds. Within each B₁₂ cluster, six weak covalent bonds persist in the *xy* plane, with an average B-B bond length of 1.90 Å. Notably, the top/bottom-most B-triangle features a bond length of 1.60 Å.



Fig. S3 Blue represents the bonds connecting each B_{12} cluster; pink indicates the six weak covalent bonds within the *xy* plane inside the B_{12} cluster; and yellow denotes the B-B bonds formed by the top/bottom B triangles within the B_{12} cluster.

References

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