Supporting Information

Automated Manufacturing of Segmented Nanowires with Thin Ferromagnetic Layers: a Step Towards Miniature SFS Josephson Junctions

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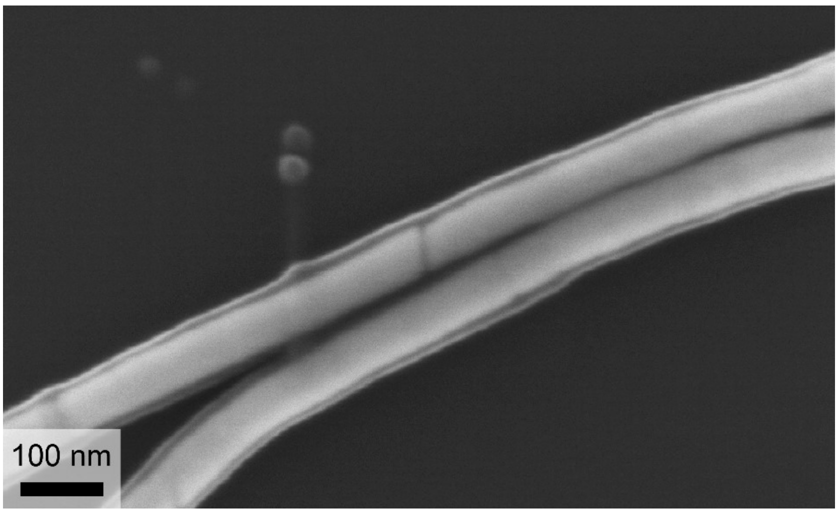

Figure S1. SEM image of individual segmented Au/Ni/Au nanowires covered by a thin layer of polyvinylpyrrolidone.

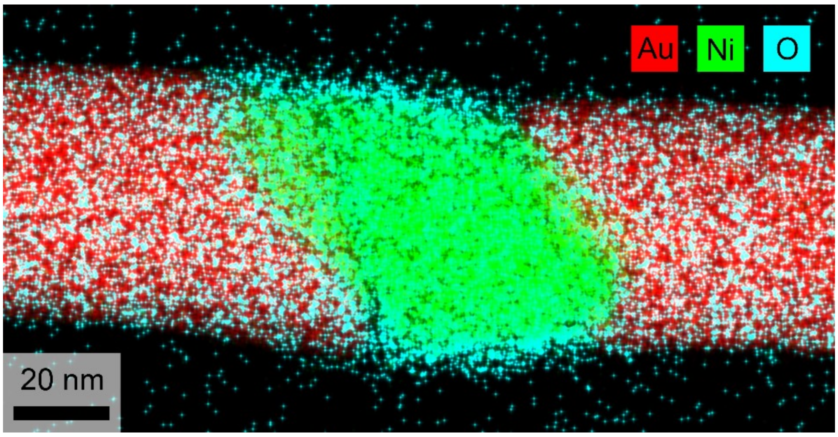

Figure S2. EDS elemental maps showing the local distribution of gold (red), nickel (green), and oxygen (blue) in Au/Ni/Au nanowires in the region of a ferromagnetic segment with a length of 46 nm. The data were obtained after a year-long storage of nanowires suspended in isopropanol.

Superconducting transport in the SNFNS bridge

We consider the superconducting properties of the SNFNS nanowire bridge within the framework of the Usadel equations[1] in a semi-one-dimensional approach, similar to Refs.[2,3] We assume that the dirty limit condition is met for all metals, the critical temperature of the weak link materials is zero, and the nanowire diameter d is much less than the coherence length of the bulk normal metal, $\xi_n = (\hbar D_n/2\pi k_B T_c)^{1/2}$, where D_n is the diffusion coefficient of normal metal N and T_c is the critical temperature of planar electrodes S.

Superconductivity in the system arises owing to the proximity effect between the superconducting electrode S and the normal layer N beneath it. The electronic structure of the N-layer regions under the S-electrode is described by the modified Usadel equation:

$$\xi_{eff}^2\frac{\partial}{\partial x}\left(G_n^2\frac{\partial\Phi_n}{\partial x}\right)-\Phi_n=-\,\delta\exp(i\chi),$$

$$\xi_{eff}^2=\frac{\xi_n^2\gamma_{BM}}{G_n(G_s+\gamma_{BM}\omega)},\ \delta=\frac{G_s\Delta}{(G_s+\gamma_{BM}\omega)}.$$

Here, Φ_n, $F_n = \Phi_{n,\omega}/(\omega^2 + \Phi_{n,\omega}\Phi^*_{n,-\omega})^{-1/2}$, $G_n = \omega/(\omega^2 + \Phi_{n,\omega}\Phi^*_{n,-\omega})^{-1/2}$, and $G_s = \omega/(\omega^2 + \Delta^2)^{-1/2}$ are the Usadel Green's functions, $\omega = (2n + 1)T/T_c$ are the Matsubara frequencies normalized by πT_c, γ_{BM} is the boundary parameter at the SN interface, x is the coordinate normalized by ξ_n, and Δ is the modulus of the order parameter in the S electrode normalized by $\pi k_B T_c$.

The region between the S electrodes is described as:

$$\xi_n^2\frac{\partial}{\partial x}\left(G_n^2\frac{\partial\Phi_n}{\partial x}\right)-\omega G_n\Phi_n=0,$$

$$\xi_f^2\frac{\partial}{\partial x}\left(G_f^2\frac{\partial\Phi_f}{\partial x}\right)-\tilde{\omega}G_f\Phi_f=0.$$

for the N and F parts of the nanowire, respectively, with boundary conditions[4] at the NF and FN interfaces

$$\pm\gamma_B\xi_nG_n\frac{d}{dx}\Phi_n=G_f\left(\frac{\omega}{\tilde{\omega}}\Phi_f-\Phi_n\right),$$

$$\mp\gamma_B\gamma\xi_fG_f\frac{d}{dx}\Phi_f=G_n\left(\frac{\tilde{\omega}}{\omega}\Phi_n-\Phi_f\right).$$

Here, $\tilde{\omega}_p = \omega + iH$, where H is the exchange energy of the F-layer normalized by πT_c. $\gamma_B = R_BA_B/\rho_n\xi_n$ and $\gamma = \rho_n\xi_n/\rho_f\xi_f$ are the suppression parameters of the interface, where ξ_n, ξ_f, ρ_n, and ρ_f represent the coherence lengths and resistivities of the N and F materials, respectively. R_B and A_B are the resistance and cross-sectional area of the corresponding interface, respectively.

The current in the N-part of the nanowire is

$$I = \frac{\pi k_B T A_B}{2e\xi_n \rho_n} \sum_{-\infty}^{\infty} \frac{G_n^2}{\omega^2} \left(\Phi_{n,-\omega}^{*} \frac{\partial \Phi_{n,\omega}}{\partial x} - \Phi_{n,\omega} \frac{\partial \Phi_{n,-\omega}^{*}}{\partial x} \right).$$

In some specific cases, the dirty limit is not applicable to the system. This can make the problem more complex and require the solution of a hybrid boundary problem involving the Usadel and Eilenberger equations.[5,6]

References

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