

## Supplementary Information

### Enhancing Magnetic Coupling in $MN_4$ -Graphene via Strain Engineering

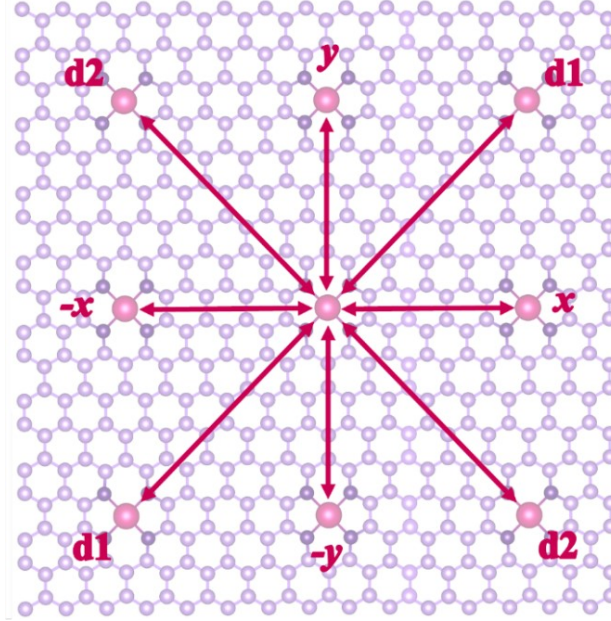
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**Fig. S1** The exchange couplings of the M atom located at the origin of a unit-cell with the neighboring atoms are shown with the purple double-head arrows along  $\pm x$ ,  $\pm y$ , and the two main diameter (d1, d2) directions.

**Table S1** The cohesive energy (CE) of the unstrained and strained  $MN_4-G$  layers. The CE defined as  $CE = \frac{[E_{MN_4-G} - (N_C E_C + N_N E_N + N_M E_M)]}{q}$ , where  $E_{MN_4-G}$  is the total energy of the studied layers, and  $E_C$ ,  $E_N$  and  $E_M$  are the total energies of free C, N, and M atoms, respectively.  $N_C$ ,  $N_N$ , and  $N_M$  are the number of C, N, and M atoms in the supercell, respectively. The parameter  $q$  stands for the total number of atoms in the supercell. For graphene, the CE per atom is -5.8275 eV.

Layer	CE/atom (eV)	Layer	CE/atom (eV)	Layer	CE/atom (eV)	Layer	CE/atom (eV)
Mn-Mn	-7.9709	Fe-Fe	-8.0017	Co-Co	-8.0198	Cu-Cu	-7.9752
Mn-Fe	-7.9698	Fe-Mn	-7.9976	Co-Mn	-8.0059	Cu-Mn	-7.9738
Mn-Co	-7.9658	Fe-Co	-8.0004	Co-Fe	-8.0043	Cu-Fe	-7.9750
Mn-Cu	-7.9730	Fe-Cu	-7.9980	Co-Cu	-8.0211	Cu-Co	-7.9725

**Table S2** Fitting parameters of  $J_r$  in Eq. (8), for MnN<sub>4</sub>-G layer under the strain.  $k_F = [(k_F^x)^2 + (k_F^y)^2]^{1/2}$  is the Fermi wave number,  $k_F^x$  and  $k_F^y$  are components of the Fermi wave vector; and  $J_0$  is a constant along  $\pm x$ ,  $\pm y$ , and the two main diameter (d1, d2) directions.

MnN <sub>4</sub> -G	Direction	$J_0(meV)$	$k_F^x(\text{\AA}^{-1})$	$k_F^y(\text{\AA}^{-1})$	$k_F(\text{\AA}^{-1})$
$\eta = -0.36$	$x$	-5.548	0.118	-----	-----
	$y$	-0.003	-----	0.002	-----
	d1	-3.939	-----	-----	0.118
	d2	-3.949	-----	-----	0.118
$\eta = -0.20$	$x$	-6.313	0.121	-----	-----
	$y$	-0.007	-----	0.002	-----
	d1	-2.210	-----	-----	0.121
	d2	-0.900	-----	-----	0.121
$\eta = 0.00$	$x$	-75.610	0.123	-----	-----
	$y$	41.040	-----	0.125	-----
	d1	-4.402	-----	-----	0.175
	d2	-1.570	-----	-----	0.175
$\eta = 0.03$	$x$	-118.600	0.150	-----	-----
	$y$	-53.000	-----	0.145	-----
	d1	-15.560	-----	-----	0.209
	d2	-8.486	-----	-----	0.209

**Table S3** Fitting parameters of  $J_r$  in Eq. (8), for FeN<sub>4</sub>-G layer under the strain. Other parameters are detailed in the caption of Table S2.

FeN <sub>4</sub> -G	Direction	$J_0(meV)$	$k_F^x(\text{\AA}^{-1})$	$k_F^y(\text{\AA}^{-1})$	$k_F(\text{\AA}^{-1})$
$\eta = -0.16$	$x$	-0.600	0.117	-----	-----
	$y$	0.361	-----	0.150	-----
	d1	-0.100	-----	-----	0.190
	d2	0.200	-----	-----	0.190
$\eta = 0.00$	$x$	-0.164	0.125	-----	-----
	$y$	-0.029	-----	0.022	-----
	d1	-0.143	-----	-----	0.127
	d2	-1.300	-----	-----	0.127
$\eta = 0.20$	$x$	-1.46	0.108	-----	-----
	$y$	-0.084	-----	0.037	-----
	d1	0.188	-----	-----	0.114
	d2	-0.310	-----	-----	0.114
$\eta = 0.23$	$x$	-0.450	0.125	-----	-----
	$y$	0.753	-----	0.091	-----
	d1	0.191	-----	-----	0.155
	d2	0.319	-----	-----	0.155

**Table S4** Fitting parameters of  $J_r$  in Eq. (8), for CoN<sub>4</sub>-G layer under the strain. Other parameters are detailed in the caption of Table S2.

CoN <sub>4</sub> -G	Direction	$J_0(meV)$	$k_F^x(\text{\AA}^{-1})$	$k_F^y(\text{\AA}^{-1})$	$k_F(\text{\AA}^{-1})$
$\eta = 0.00$	$x$	-0.004	0.129	-----	-----
	$y$	-0.0004	-----	0.029	-----
	d1	-0.001	-----	-----	0.132
	d2	-0.001	-----	-----	0.132
$\eta = 0.16$	$x$	48.460	0.223	-----	-----
	$y$	0.099	-----	0.097	-----
	d1	0.240	-----	-----	0.243
	d2	-3.721	-----	-----	0.243
$\eta = 0.36$	$x$	0.833	0.099	-----	-----
	$y$	-0.100	-----	0.046	-----
	d1	0.109	-----	-----	0.109
	d2	-0.109	-----	-----	0.109
$\eta = 0.39$	$x$	19.65	0.221	-----	-----
	$y$	0.131	-----	0.111	-----
	d1	0.047	-----	-----	0.247
	d2	-1.391	-----	-----	0.247

**Table S5** Calculated ( $k_F^{Cal}$ ) and fitted ( $k_F^{Fit}$ ) Fermi wave numbers for various strained and unstrained MN<sub>4</sub>-G layers under the strain.

MnN <sub>4</sub> -G			FeN <sub>4</sub> -G			CoN <sub>4</sub> -G		
$\eta$	$k_F^{Cal}(\text{\AA}^{-1})$	$k_F^{Fit}(\text{\AA}^{-1})$	$\eta$	$k_F^{Cal}(\text{\AA}^{-1})$	$k_F^{Fit}(\text{\AA}^{-1})$	$\eta$	$k_F^{Cal}(\text{\AA}^{-1})$	$k_F^{Fit}(\text{\AA}^{-1})$
-0.36	0.112	0.118	-0.16	0.106	0.190	0.00	0.105	0.132
-0.20	0.100	0.121	0.00	0.120	0.127	0.16	0.149	0.243
0.00	0.125	0.175	0.20	0.101	0.114	0.36	0.139	0.109
0.03	0.137	0.209	0.23	0.122	0.155	0.39	0.132	0.247