Supporting Informations for

Predictive of the Quantum Capacitance Effect on the Excitation of Plasma Waves in Graphene Transistors with Scaling Limit

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S1. The optical conductivity of graphene based on the Kubo theorem

To describe the transport properties of graphene under alternative electromagnetic field in our study, we calculate the local conductivity of graphene from Kubo formula[1,2]. The conductivity calculated consists of two parts, the intraband σ_{intra} and interband part σ_{inter}

$$\sigma_{\mathrm{Gr}}(\omega)=\frac{\mathrm{e}^2}{i\pi\hbar^2}\left[\frac{1}{\omega+i\tau^{-1}}\int_0^\infty \varepsilon\left(\frac{\partial F(\varepsilon)}{\partial\varepsilon}-\frac{\partial F(-\varepsilon)}{\partial\varepsilon}\right)d\varepsilon\right]-\int_0^\infty\frac{F(-\varepsilon)-F(\varepsilon)}{(\omega+i\tau^{-1})^2-4(\varepsilon/\hbar)^2}d\varepsilon \qquad (1)$$

where $F(\varepsilon)=\{1+\exp[(\varepsilon-E_F)/k_BT]\}^{-1}$ is the Fermi-Dirac distribution with E_F as the Fermi-level, k_B and T are the Boltzman constant and temperature, respectively. The first term corresponds to contributions from intraband electron-phonon scattering, and the second term arises from contribution due to direct interband electron transitions. The difference of the Fermi functions in the second integrand reads

$$G(\varepsilon)=\frac{\sinh(\varepsilon/k_BT)}{\cosh(\varepsilon/k_BT)+\cosh(E_F/k_BT)} \qquad (2)$$

Integration of the first term leads to

$$\sigma^{\mathrm{intra}}=\frac{2i\mathrm{e}^2k_BT}{\pi\hbar^2(\omega+i\tau^{-1})}\ln\left[2\cosh\left(\frac{E_{\mathrm{F}}}{2k_BT}\right)\right] \qquad (3)$$

For the interband term, we obtain

$$\sigma^{\mathrm{inter}}=\frac{\mathrm{e}^2}{4\hbar}\left[G(\hbar\omega/2)-\frac{2\hbar\omega}{i\pi}\times\int_0^\infty\frac{G(\varepsilon)-G(\hbar\omega/2)}{(\hbar\omega)^2-4\varepsilon^2}d\varepsilon\right] \qquad (4)$$

It should be noted that the frequency of plasma resonance is mainly determined by the imaginary part of conductivity, while the real part of the conductivity causes the energy loss and broadening of plasma resonance. With the help of Eq. 4 and for $E_F \gg k_B$T, we the intra- and inter-band conductivity reads:

$$\sigma^{\mathrm{intra}}=\frac{i\mathrm{e}^2E_F}{\pi\hbar^2(\omega+i\tau^{-1})},\quad \sigma^{\mathrm{inter}}=\frac{\mathrm{e}^2}{4\hbar}\left[1+\frac{i}{\pi}\ln\frac{\hbar(\omega+i\tau^{-1})-2E_F}{\hbar(\omega+i\tau^{-1})+2E_F}\right] \qquad (5)$$

Considering the graphene monolayer is surrounded with dielectrics of constants ε_1 (top) and ε_2

(bottom), for definiteness, $\varepsilon_1 \sim \varepsilon_1(q)$ as a function of plasma wavevector q is assumed considering the situation when the gates are added on top of the graphene (seeing Fig. 2a). Electric potential is found from Poisson-equation if the gate contact is separated from the graphene with dielectric layer ε_1 thickness d, the boundary conditions is a zero potential at contact and continuity condition of electric flux density at graphene interface. The Poisson equation can be solved in the Fourier form, which results in following relations

$$\varphi(\mathbf{q},\ \mathbf{d}) = \frac{e}{\varepsilon_2 q}\exp(\text{-qd})\left[1 + \frac{\varepsilon_2 - \varepsilon_1 \coth qd}{\varepsilon_2 + \varepsilon_1 \coth qd}\exp(\text{2qd})\right] \quad (6)$$

In our investigated regime, $qd \ll 1$, Eq. (6) takes form as follows

$$\varphi(\mathbf{q},\ \mathbf{0}) = \frac{2e}{q(\varepsilon_2 + \varepsilon_1 \coth(\text{qd}))} \quad (7)$$

while $\varphi(q,0) \sim \dfrac{2e}{q(\varepsilon_1 + \varepsilon_2)}$ along the graphene plane when the gate is absent.

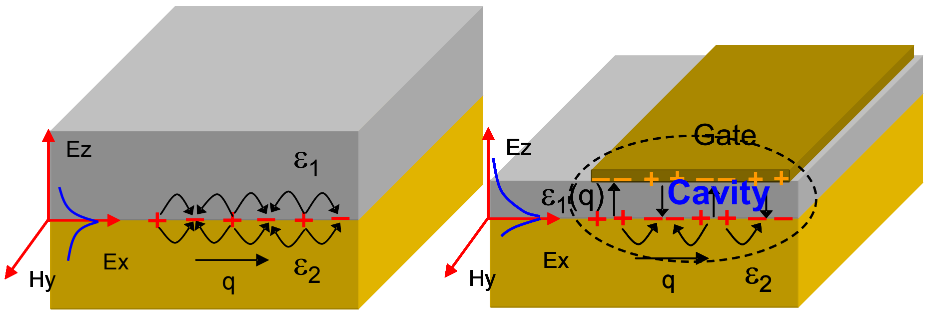

Fig. S1 (left) Plasma wave in pristine graphene sheet, (right) plasma oscillation in the cavity formed by the gate and channel.

Now, we consider the geometry and the longitudinal plasma wave in Fig. S1. The electric field takes form $E_{z1,2} = A_{1,2}e^{iqx - Q_{1,2}z}$, $E_y = 0$, $E_{x1,2} = B_{1,2}e^{iqx - Q_{1,2}z}$, where $Q_{1,2} \sim \sqrt{q^2 - \frac{\varepsilon_{1,2}\omega^2}{c^2}}$, after matching the boundary conditions, the dispersion of plasma wave is determined by $q \approx 2\varepsilon \dfrac{i\omega}{\sigma(\omega)}$,

where $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$ and $\varepsilon = (\varepsilon_2 + \varepsilon_1 \coth(qd))/2$ for the graphene with and without gate, and the plasma frequency is determined by the imaginary part of the intra-conductivity. From Eqs.1-5 it is straightforward to obtain plasma dispersion relation:

$$\omega = \sqrt{\frac{e^2 E_F q}{2\pi\hbar^2 \varepsilon}},\ \varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2} \text{ for ungated graphene,}$$
$$\varepsilon = (\varepsilon_2 + \varepsilon_1 \coth(qd))/2, \text{ for gated graphene} \quad (8)$$

Since $\coth(qd) \sim 1/qd$, when $qd \ll 1$, the plasma frequency has a linear relation with wavevector q.

S2. Random Phase Approximation

The frequency dependence of dynamical dielectric function for two-dimensional electron system is given by $\varepsilon(\mathrm{q}, \omega)/\kappa=1-e^2/2\kappa \mathrm{q}\Pi(\mathrm{q}, \omega)$[3, 4], where κ is the effective dielectric constant determined by geometry of the structure. Π is polarizability function, which is given by the bubble diagram

$$\Pi(\mathbf{q}, \omega)=g\sum_{ss'}\frac{f_{\mathbf{k}}^{s}-f_{\mathbf{k+q}}^{s'}}{\omega+\varepsilon_{\mathbf{k}}^{s}-\varepsilon_{\mathbf{k+q}}^{s'}}F_{ss'}(\mathbf{k},\ \mathbf{k+q}) \tag{9}$$

in which $f^s_{\mathrm{k}}=[\exp\{\beta(\varepsilon^s_{\mathrm{k}}-\mu)\}+1]^{-1}$ is the Fermi distribution function, β is thermal energy, μ is chemical potential, g is degeneracy factor (g is 4), and $F_{ss'}$ is the wave-function overlap integral ($F_{ss'}=[1+ss'\cos(2\theta)]/2$), with θ being the angle between k and k+q. By looking for the zeros of dynamical polarizability in the long-wavelength limit (q→0), the polarizability approximately follows $\Pi(q,\ \omega)\approx\frac{D_0 v_F^2 q^2}{2\omega^2}\left[1-\frac{\omega^2}{4E_F^2}\right]$, and thus results in the plasma wave dispersion $\omega(\mathrm{q})\sim(ge^2E_F/8\pi\hbar^2\kappa)$, κ is relevant with the surrounding dielectrics, and in consistent with above discussions in Eq. 8.

S3. Plasma analysis based on quasi-classical Boltzman theory

The starting point of Boltzman transport theory is to search for the distribution function due to the inter-carrier collisions and the disorder. Recent optical-pump THz-probe experiment has confirmed the fast relaxation of photo-excited electron-hole pairs in a few tens of fs, which is much shorter than the frequency of plasma wave ranging from THz to mid-infrared wavelength. It is well known that the Fermi-distribution of electron and hole turns the collision integral into zero, and thus it is ensured to allow the quasi-equilibrium treatment of Dirac fermions system. For the sake of completeness, in this Supporting Information, we summarize the hydrodynamic treatment for graphene, starting from quasi-classical Boltzman transport equation:

$$\frac{\partial F}{\partial \mathrm{t}}+v\cdot\nabla_r F+e\nabla U\cdot\nabla_p F=\left(\frac{\partial F}{\partial \mathrm{t}}\right)_{ee} \tag{10}$$

Here, F is the distribution function of graphene representing the probability to find an electron with momentum p. As known, the Fermi distribution function turns the right term of Eq.10 into zero. On integrating the Bolzman kinetic equation over phase space, the continuity equation and Euler equation can be obtained:

$$\frac{\partial n}{\partial t}+\frac{\partial nv}{\partial x}=0\,,\qquad \frac{\partial p}{\partial t}+v\cdot\frac{\partial p}{\partial x}+e\frac{\partial U}{\partial x}+\frac{p}{\tau}=0 \tag{11}$$

Where p is a hydrodynamic momentum, n is the electron density, U is the potential along the channel, v is the electron velocity, τ is the momentum relaxation time. In the 2D massless Dirac fermion case, one finds $p=3Pv/nv_F^2$, v_F is the Fermi velocity, and the term P is the hydrodynamic pressure given by

$$P=\frac{v_F<p>}{2}=k_BTn\frac{F_2(\xi)}{F_1(\xi)},\ \text{and}\ F_n(\xi=E_F/k_BT)=\int_0^\infty dx\frac{x^n}{1+\exp(x-\xi)} \tag{12}$$

and $<p>=\int_0^{\infty}(1+\exp((\varepsilon_p(v_F p)-\mu)/k_B T)^{-1} 2p^2/\pi\hbar^2 dp$ is the momentum modulus per unit area. The fictitious mass of Dirac fermions is given by $M_e \sim 3<p>/2v_F$, when $E_F >> k_B T$, M_e is reduced to cyclotron mass $\sim E_F/v_F^2$. To derive the spectra of plasma waves, one can apply small signal analysis to the first order (i. e. $E=E_0+E_1 e^{iqx-i\omega t}$, $v=v_0+v_1 e^{iqx-i\omega t}$, and $n=n_0+n_1 e^{iqx-i\omega t}$). In the graphene FET with gate dielectric layer, the carrier density is related to the gate potential as in Eq. 1$\sim n=C_{TG}V_{TG}$. And, with $U(q)=E/iq$ and $n\sim C_{TG}E/iq$, from the linearized Euler equation and continuity equation to the first order, the following relationship can be obtained

$$-i\frac{3}{2}\omega\frac{<p>}{v_F}+\frac{iq^2n^2}{\omega}\left(\frac{v_F}{<p^{-1}>}+\frac{e^2}{C_{TG}}\right)+\gamma=0$$

where $<p>\sim 2nM_e v_F/3$, and $<p^{-1}>\sim D_0 v_F$ and D_0 is the density of state. Thus the plasma wave frequency is obtained as follows

$$\omega\sim\sqrt{\frac{q^2 n(1+r_e)}{M_e D_0}},\ \text{where } r_e \text{ is } \frac{D_0 e^2}{C_{TG}}$$

To estimate the magnitude of r_e (seeing the text), it is known that D_0 is in the order of 10^{17}m^2/eV, C_{TG} is in the order of 10^{-6}F/cm^2, and thus r_e is much larger than 1. For the density of state in the perfect graphene sheet ($D_0=2E_F/\pi\hbar^2$), the frequency of plasma wave is given by $\omega=\sqrt{\frac{ne^2d}{M_e\varepsilon_1}}q$, in well consistent with Eq. 8 in the above section when considering the screening effect of gate and the effective dielectric constant ε of Eq. 8 is reduced to $\frac{1}{2}[\varepsilon_2+\varepsilon_1\coth(qd)]$, with $\coth(qd)\sim 1/qd$ when $qd<<1$. Therefore, the RPA approximation, Kubo theory and Boltzman transport methods can lead to the similar results in the long-wavelength limit.

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