Electronic Supplementary Material (ESI) for RSC Advances

"Spring-type piezoelectric energy harvester"

Dongjin Kim, Seungbum Hong,* Dongjun Li, Hee Seok Roh, Gun An, Jiyoon Kim, Moonkyu Park, Jongin Hong, Tae-hyun Sung and Kwangsoo No*

Calculation of pure piezoelectric signal via Fourier transform

We calculated the amplitude of pure piezoelectric voltage signal generated by the energy harvester via removing the noise signal using Fourier transform. The functions of the spring length variation (reference signal) and pure piezoelectric voltage (response signal) are expressed as follows.

$$\text{Reference signal} = V_R \sin(\omega_R t+\theta_R) \qquad (1)$$

$$\text{Response signal} = V_I \sin(\omega_R t+\theta_I) \qquad (2)$$

To figure out V_I, we had performed the following calculations. First, the output voltage signal was multiplied by the reference signal.

$$V_{M1} = V_I V_R \sin(\omega_R t+\theta_I)\sin(\omega_R t+\theta_R)$$

$$= \tfrac{1}{2} V_I V_R \cos(\theta_R-\theta_I)+\tfrac{1}{2} V_I V_R \sin(2\omega_R t+\theta_R+\theta_I) \qquad (3)$$

Then, we integrated V_{M1} from 0 to 10π as the full time length corresponded to five times the period

$$\int_0^{10\pi} [\frac{1}{2} V_I V_R\, cos(\theta_R-\theta_I)+\frac{1}{2} V_I V_R\, sin(2\omega_R t+\theta_R+\theta_I)]dt$$

$$= 5\pi V_I V_R\, cos(\theta_R-\theta_I) \qquad (4)$$

* Corresponding authors: hong@anl.gov and ksno@kaist.ac.kr

$$V_{M1+FILT} = 5\pi V_I V_R \cos(\theta_R - \theta_I) \quad (5)$$

In order to remove $\cos(\theta_R - \theta_I)$, the output voltage signal was multiplied by the another reference signal, $V_R\sin(\omega_R t+\theta_R-\pi/2)$ that is 90° out of phase with respect to the original reference signal, $V_R\sin(\omega_R t+\theta_R)$.

$$V_{M2} = V_I V_R \sin(\omega_R t+\theta_I)\sin(\omega_R t+\theta_R-\pi/2)$$

$$= ½V_I V_R\cos(\theta_R-\theta_I-\pi/2) + ½V_I V_R\sin(2\omega_R t+\theta_R+\theta_I-\pi/2) \quad (6)$$

$\frac{1}{2}V_I V_R\sin(2\omega_R t+\theta_R+\theta_I-\pi/2)$ was also removed by integrating Eq. (6) with respect to time from 0 to 10π.

$$\int_0^{10\pi} [\frac{1}{2} V_I V_R \cos(\theta_R - \theta_I - \frac{\pi}{2}) + \frac{1}{2} V_I V_R \sin(2\omega_R t + \theta_R + \theta_I - \frac{\pi}{2})] dt$$

$$= 5\pi V_I V_R \cos(\theta_R - \theta_I - \frac{\pi}{2}) \quad (7)$$

$$V_{M2+FILT} = 5\pi V_I V_R\cos(\theta_R-\theta_I-\pi/2) = 5\pi V_I V_R\sin(\theta_R-\theta_I) \quad (8)$$

Finally, we were able to obtain the amplitude of pure piezoelectric voltage signal (V_I).

$$V_I = (\frac{1}{5\pi}V_R)\sqrt{V_{M1+FILT}^2 + V_{M2+FILT}^2} \quad (9)$$

Using Eq. (9), we obtained 20.62 mV as the amplitude of piezoelectric voltage signal (V_I). Figure S1 shows the output signal generated from the energy harvester and the pure piezoelectric output calculated by the Fourier transform.

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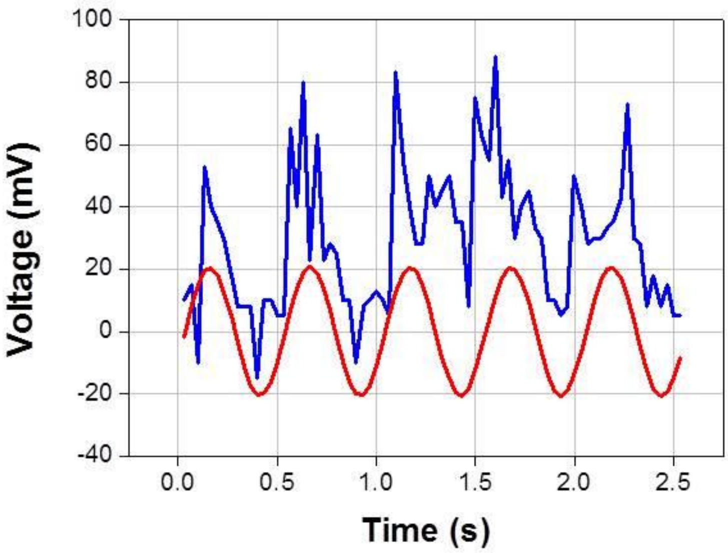

Figure S1. The output signal generated from the energy harvester (blue line) and the pure piezoelectric output calculated by Fourier transform (red line).

Calculation of the effective piezoelectric coefficient

If we simplify the spring structure into a cylindrical rod as shown in Figure S2, we can use Gauss' law to calculate the total charge accumulated on one electrode based on the capacitance and output voltage. Eq. (10) describes the electric field in the radial direction (r) inside the ferroelectric layer (E_r) in terms of the total charge (Q), permittivity ($\varepsilon_0\varepsilon_r$) and size of the cylindrical rod (a: inner diameter, b: outer diameter, and L: length).

$$2\pi r L E_r = \frac{Q}{\varepsilon_0 \varepsilon_r}, E_r = \frac{Q}{\varepsilon_0 \varepsilon_r 2\pi r L} \qquad (10)$$

Figure S2. Schematic of a cylindrical rod covered by ferroelectric layer and surface electrode.

Integrating the E_r from inner radius, a, to outer radius, b, of the cylindrical rod, we obtain the output voltage generated by the rod.

$$\int_a^b E_r\,dr = \int_a^b \frac{Q}{\varepsilon_0 \varepsilon_r 2\pi r L} dr, \int_a^b E_r\,dr = V_I = 20.62\ mV \quad (11)$$

By equating the measured output voltage of 20.62 mV with Eq. (11), we can calculate Q as follows:

$$Q = \varepsilon_0 \varepsilon_r 2\pi L \times \frac{20.62mV}{ln\frac{b}{a}} = \frac{11 \times 8.85 \times 10^{-12} F/m \times 2\pi \times 0.055\pi\ m \times 0.02062V}{ln\frac{0.49143}{0.485}}$$

$$= 1.65 \times 10^{-10}\ C \quad (12)$$

Then we calculated the half of the peak to valley amplitude of applied force to the spring structure using Hooke's law, where the peak to valley amplitude of displacement was 15.92 mm.

$$F = 725.9\ N/m \times 0.01592\ m\ /2 = 5.78\ N, \quad (13)$$

From Eqs. (12) and (13), we obtained the effective piezoelectric coefficient.

$$d_{33,eff} = Q/F = 1.65 \times 10^{-10}\ C/5.78\ N = 28.55\ pC/N \quad (14)$$

Measurement of the noise component of energy harvester

In order to ensure that Fig. 3c contains real piezoelectric response, the same measurement was conducted using non-ferroelectric polymer coated spring-type devices. Figure S3A shows actual image of the non-ferroelectric polymer coated device which is basically the same structure with the energy harvester except the non-ferroelectric polymer layer inserted between the spring and surface electrode (gold coated by PVD on the polymer). We used ethyl 2-cyanoacrylate as non-ferroelectric material, which is commercially available super glue (LOCTITE®).

Figure S3B shows the noise signal of the non-ferroelectric polymer coated spring-type device. We collected the signal using the digital oscilloscope when the spring was cyclically compressed 12 times during 5 second. The noise signal of the device was different with that of the piezoelectric energy harvester and the maximum noise signal was 17.5 mV.

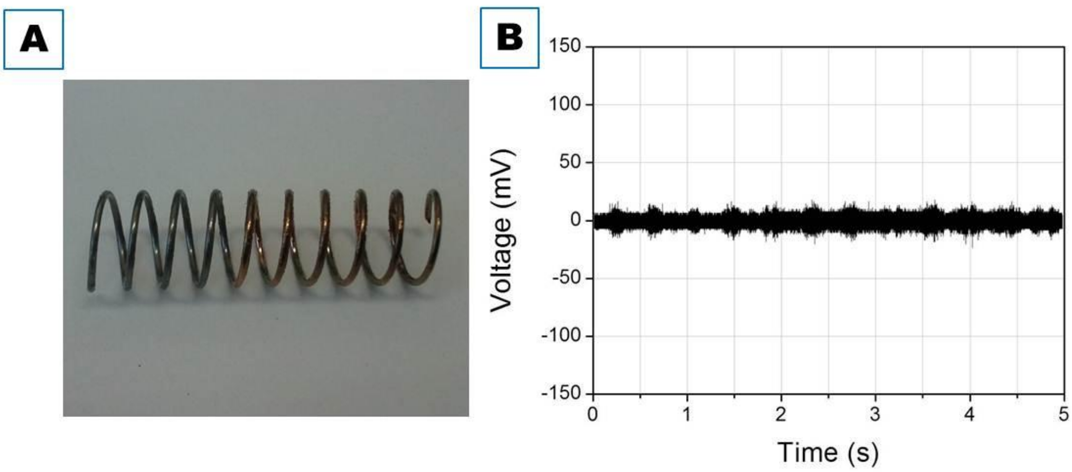

Figure S3. (A) Actual image of the non-ferroelectric polymer coated device, and (B) the noise signal generated by the device.

Finite Element Analysis on a spring coated with P(VDF-TrFE) film

In order to investigate the stress distributions, finite element analysis was performed on a spring coated with P(VDF-TrFE). Material properties used in the FE analysis are listed in Table S1.

Table S1. Mechenical Properties of steel and P(VDF-TrFE)

	E (GPa)	ν	ρ (kg/m^3)
Steel	200	0.32	7860
P(VDF-TrFE)	8.3	0.18	1575

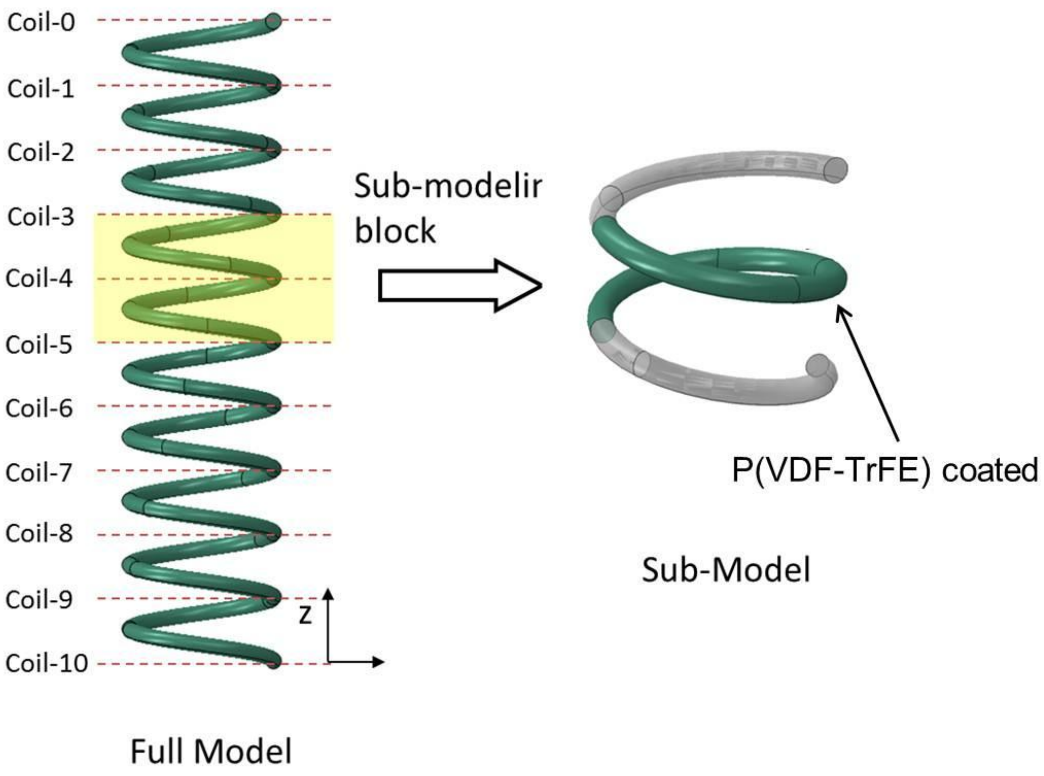

Figure S4. Schematic diagrams of spring structures used in full model (left) and sub-model (right) analysis.

As shown in Figure S4, the FE analysis consists of two parts: full model analysis and sub-model analysis. In the full model analysis, the spring was created using 3-dimensional elements but the P(VDF-TrFE) film was not considered since its stiffness is quite less compared to spring steel. However, in the sub-model analysis the P(VDF-TrFE) film was modelled using shell elements of dimension mostly 0.1 mm × 0.1 mm. One end of the spring was fixed while the other end was allowed to translate only along the loading direction by constraining other degrees of freedom. A vertical load of 17.4 N was applied on the free end.

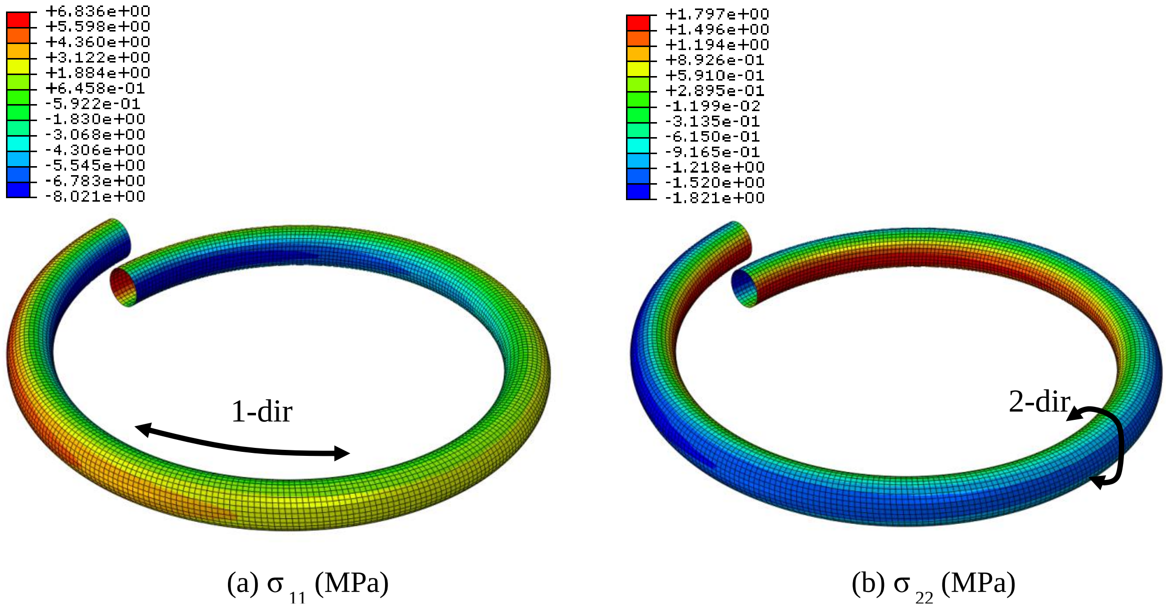

Figure S5. Stress distributions on P (VDF-TrFE) shell elements (a) σ_{11} (b) σ_{22}.

From the full model analysis, for the applied load, the spring displacement was measured to be 26.2 mm, about 9% higher than the experimental result of 24.0 mm. Figure S5 shows stress distributions on P(VDF-TrFE) film coated on the spring. It is shown that when the spring is loaded in compression, stress at the outer surface of the spring is in compression in 2-direction but in tension in 1-direction, where 1- and 2-

directions indicate circumferential and hoop directions as shown in Fig. 4. With thin shell theory, it is accepted that electric displacement, q (Coulomb/m^2) for coated P(VDF-TrFE) film is related only to the normal stresses (σ_{11} and σ_{22}) which are the components of in-plane stresses in shell elements, and it is independent of shear stress σ_{12}. Therefore, d_{31} is the main factor on piezoelectric behaviour of the spring-type energy harvester. For the case of using a poled piezoelectric material whose d_{31} is 49 pC/N, it is expected that the effective piezoelectric constant is 383 pC/N for springs with 10 turns. It means that the spring structure coated by P (VDF-TrFE) film with 10 turns can amplify the effective load by about 7.8 times to generate piezoelectric charges, leading to 7.8 times larger effective piezoelectric coefficient. However, as we only coated 5.5 turns in our experiment, the amplification factor is expected to reduce to 4.3 times.

Calculation of resonance frequency and spring constant

The resonance frequency of the spring is found to be,

$$f_{res} = \frac{d}{9D^2 n_t}\sqrt{\frac{G}{\rho}} \tag{15}$$

where d is the wire diameter, D is the nominal coil diameter, n_t is the total number of coils, G is the shear modulus and ρ is density of the material, respectively. The resonance frequency of the spring can be easily tuned via changing the shape of the spring, i.e. the wire diameter or the coil diameter.

There is another method to calculate the resonance frequency of the spring structure, which we used for our experiment. We calculated the resonance frequency of the spring using the spring constant, which was measured by placing a mass of 500 g at the end of the spring as follows [1].

$$\mathrm{k} = \frac{F}{x} = \frac{mg}{x} \tag{16}$$

$$\mathrm{k} = \frac{0.5\ kg \times 9.8\ m/s^2}{0.00675} = 725.9\ \mathrm{N/m} \tag{17}$$

The resonance frequency of the spring was calculated as follows.

$$f_{res} = \frac{1}{2}\sqrt{\frac{k}{M}} \tag{18}$$

$$f_{res} = 0.5 \times \sqrt{\frac{725.9\ N/m}{0.00222\ Kg}} = 285.9\ Hz \tag{19}$$

Analysis of crystalline phase of P(VDF-TrFE) films on the spring structure

In order to compare the crystalline phases and the structures of the polymer chains in P(VDF-TrFE) films deposited on the spring and the iron plate (model system), we further conducted FTIR and XRD analysis of P(VDF-TrFE) films deposited on the spring structures. Figure S6A shows XRD pattern of P(VDF-TrFE) film on the spring structures. P(VDF-TrFE) film showed a peak in intensity at 2θ = 19.91°, which stems from the reflections of (110) and (200) crystal planes. When compared with the XRD pattern on iron plate (See Fig. 2b), the peak shift of 0.04° was observed, which may result from the stress generated due to the coating on different geometry of surfaces. In the FTIR spectra (Fig. S6B), the absorption bands at 844, 880, 1076, 1119, 1170, 1287, 1400 and 1430 cm^{-1} were observed, which is the same as the IR spectra of P(VDF-TrFE) deposited on the iron plate. Although the peak shift slightly appeared in XRD pattern, both XRD and FTIR results showed similar results with those observed on the iron plate, as shown in Fig. 2b and 2c.

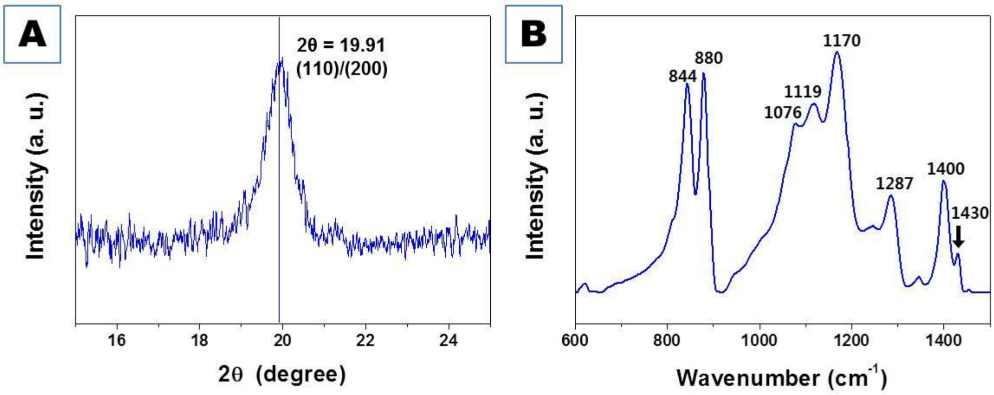

Figure S6. (A) XRD pattern and (B) FTIR spectra of P(VDF-TrFE) films deposited on the spring structure

References:

[1] See, e.g. http://www.efunda.com/DesignStandards/springs/spring_frequency.cfm