Electronic Supplementary Material (ESI) for RSC Advances.

Supporting Information

Length evolution of helical micro/nano-scale structures

Yuanqiang Song[1†], Xian Jian[1†], Yu Pan[1†], Bo Wang[2], Weirong Huo[1], Aifang Liu[1], Weiqiang Lv[1] & Weidong He[1,3,4*]

[1]School of Energy Science and Engineering, University of Electronic Science and Technology of China (UESTC), Chengdu, 611731, PR China, [2]School of Mathematics, University of Electronic Science and Technology of China (UESTC), Chengdu, 611731, PR China, [3]Interdisciplinary Program in Materials Science, Vanderbilt University, Nashville, Tennessee 37234-0106, USA, [4]Vanderbilt Institute of Nanoscale Science and Engineering, Vanderbilt University, Nashville, Tennessee 37234-0106, USA. *Correspondence and requests for materials should be addressed to W. H. (email: weidong.he@uestc.edu.cn and weidong.he@vanderbilt.edu). †These authors contributed equally to this work.

The dependence of L on h in both CLHL and COHL cases.

Both L_{CLHL} and L_{COHL} increase linearly with increasing *h*, as shown in Figures S1 and S2. The plots show a non-linear dependence of *L* on *d*. Just as mentioned in the manuscript, there exists a critical point for *d*, below which the increasing slope of *L* with increasing h is much slower; above the critical point of *d*, the increase becomes much steeper with the increase of *h*.

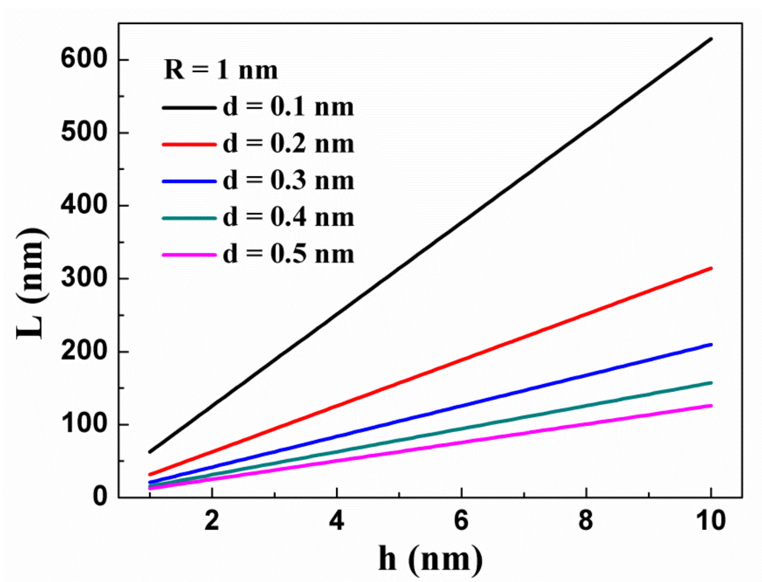

Figure S1 | Plots of L_{CLHL} vs. *h* with fixed *R* and *d*.

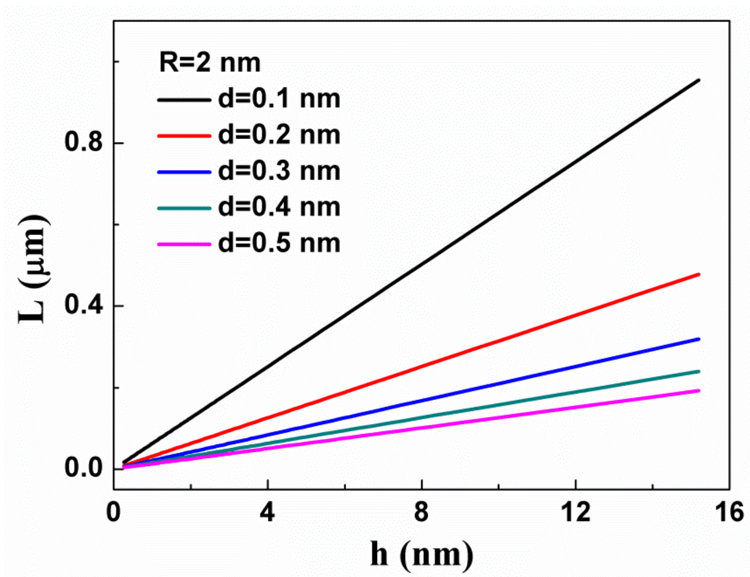

Figure S2 | Plots of L_{COHL} vs. h with fixed R and d.

Derivation of the helical length formula of a COHL structure

The equation of the COHL can be parameterized using the rotation angle θ in a cylindrical coordinate. Therefore, the parametric equation of the COHL can be described in Eq. 1,

$$\begin{cases} x(\theta) = R(\theta)\cos(\theta) \\ y(\theta) = R(\theta)\sin(\theta)\,, \quad 0 \le \theta \le 2k\pi \\ z(\theta) = v\theta \end{cases} \qquad (1)$$

where $v = \dfrac{d}{2\pi}$ and $R(\theta) = R - v\theta\cot\alpha$.

The helical length of the COHL (L_{COHL}) can be obtained through the curve integral as exhibited in Eq. 2.

$$L_{COHL} = \int_0^{2k\pi} \left\{[x'(\theta)]^{2\prime} + [y(\theta)]^2 + [z(\theta)]^2\right\}^{1/2} d\theta \qquad (2)$$

Substituting Eq.1 into Eq. 2, we then obtain Eq. 3,

$$L_{COHL} = v\int_0^{2k\pi} [\theta^2 \cot^2\alpha - \theta\cot\alpha \frac{2R}{v} + \mathrm{c}]^{1/2} d\theta$$

(3)

where $k = \dfrac{h}{d}$ and $c = 1 + \cot^2\alpha + \left(\mathrm{R}/v\right)^2$.

To simplify the derivation, we denote the coefficient $\cot^2\alpha$ as a, $(2R/v)\cot\alpha$ as b and the integrand $(a\theta^2 + b\theta + c)^{1/2}$ as F. Therefore, Eq. 3 can be solved through the following integrals in Eqs. 4 and 5 by assuming $a > 0$ and $4ac - b^2 > 0$.

$$\int F d\theta = \frac{2a\theta - 3b}{4a^2} F + \frac{3b^2 - 4ac}{8a^2} \int \frac{d\theta}{F} \tag{4}$$

$$\int \frac{d\theta}{F} = \frac{1}{\sqrt{a}} \operatorname{arsinh} \frac{2a\theta + b}{\sqrt{4ac - b^2}} \tag{5}$$

Substituting Eq. 4 into Eq. 5, we get Eq. 6.

$$\int F d\theta = \frac{2a\theta - 3b}{4a^2} F + \frac{3b^2 - 4ac}{8a^{2.5}} \operatorname{arsinh} \frac{2a\theta + b}{\sqrt{4ac - b^2}}$$

(6)

By replacing a, b, c and F in Eq. 6 with the original form, we finally obtain the helical length L_{COHL} as shown in Eq. 7.

$$L_{COHL} = \left[\frac{v\theta cot\alpha - R}{2cot\alpha}\sqrt{(\theta cot\alpha - \frac{R}{v})^2 + 1 + cot^2\alpha} + \frac{v(1 + cot^2\alpha)}{2cot\alpha} arsinh(\frac{v\theta cot\alpha - R}{v\sqrt{1 + cot^2\alpha}})\right]\Bigg|_0^{2k}$$

(7)

Simplifying the result by replacing $R - v\theta\cot\alpha$ with $\mathrm{R}(\theta)$, we get Eq. 8.

$$L_{COHL} = \left[\frac{-R(\theta)}{2cot\alpha}\sqrt{\frac{R^2(\theta)}{v^2} + 1 + cot^2\alpha} + \frac{v(1 + cot^2\alpha)}{2cot\alpha} arsinh(\frac{-R(\theta)}{v\sqrt{1 + cot^2\alpha}})\right]\Bigg|_0^{2k\pi} \tag{8}$$

By replacing v with $v = d/2\pi$, and $\mathrm{R}(\theta)$ with $R - v\theta\cot\alpha$, we get the final form of the helical length L_{COHL}, written as Eq. 9, where $k = \dfrac{h}{d}$.

$$L_{COHL} = \left[\frac{\theta dcot\alpha - R}{4\pi cot\alpha}\sqrt{\frac{(4\pi^2 R - dv\theta cot\alpha)^2}{d^2} + 1 + cot^2\alpha} + \frac{d(1 + cot^2\alpha)}{4\pi cot\alpha} arsinh(\frac{d(\theta dco}{4\pi^2\sqrt{1}}\right.$$

(9)